

# “SOLVING” QUADRATIC EQUATIONS

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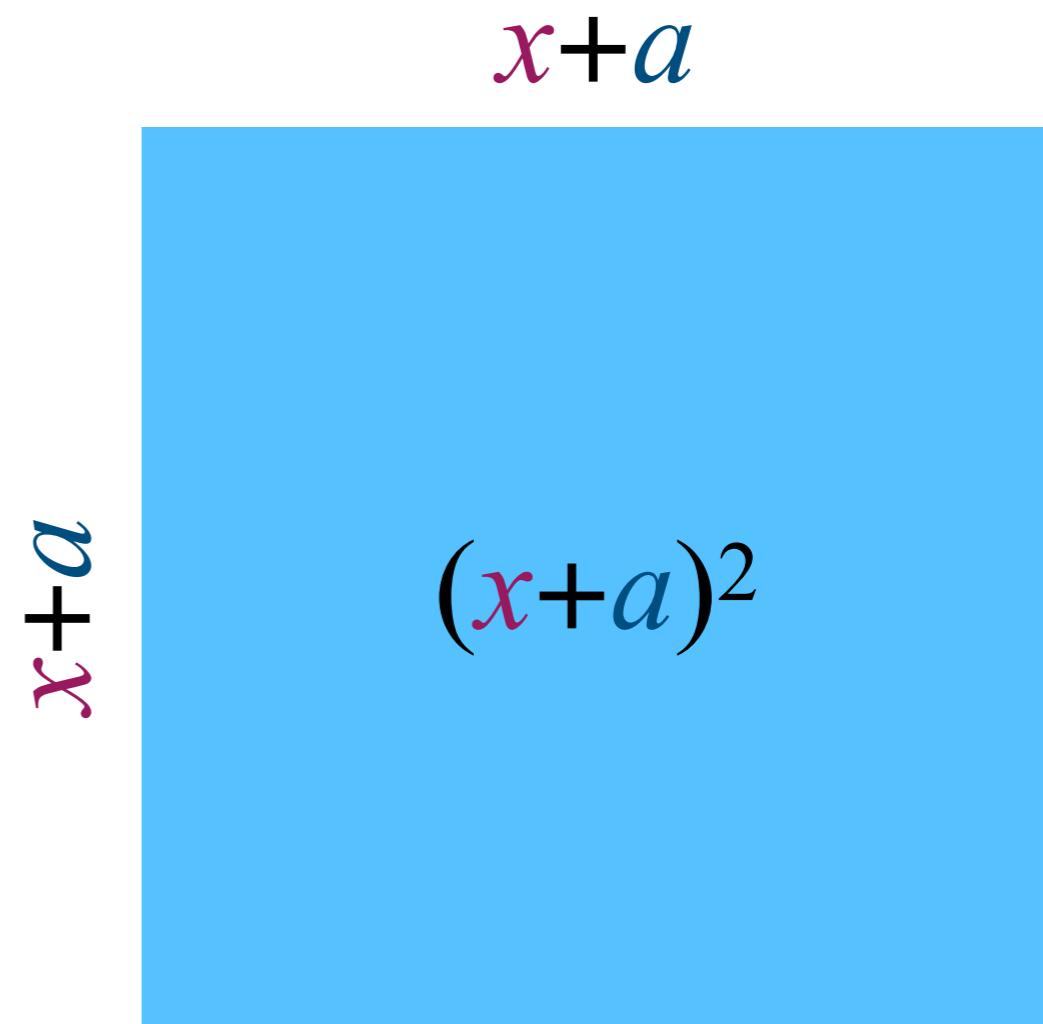
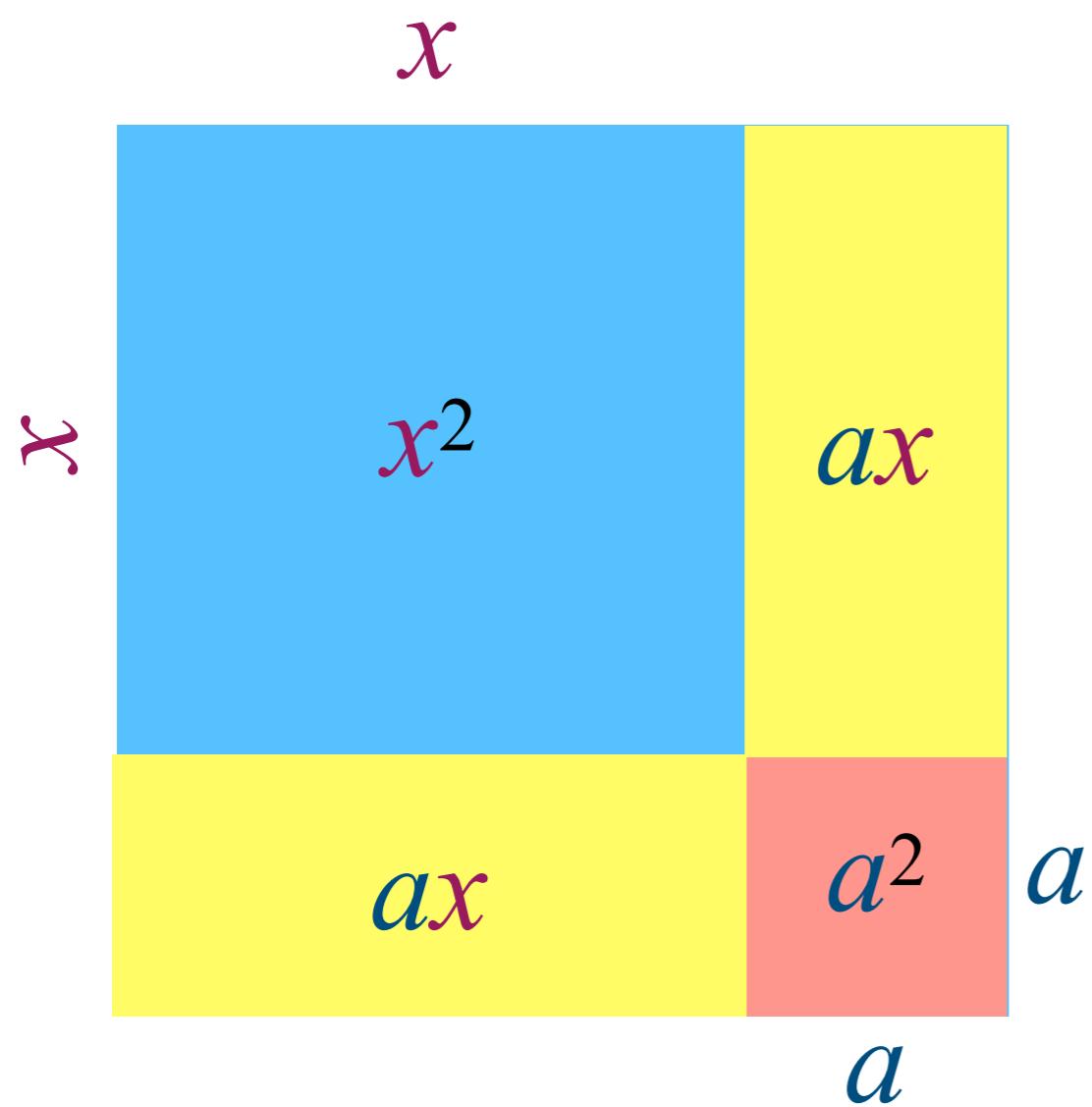
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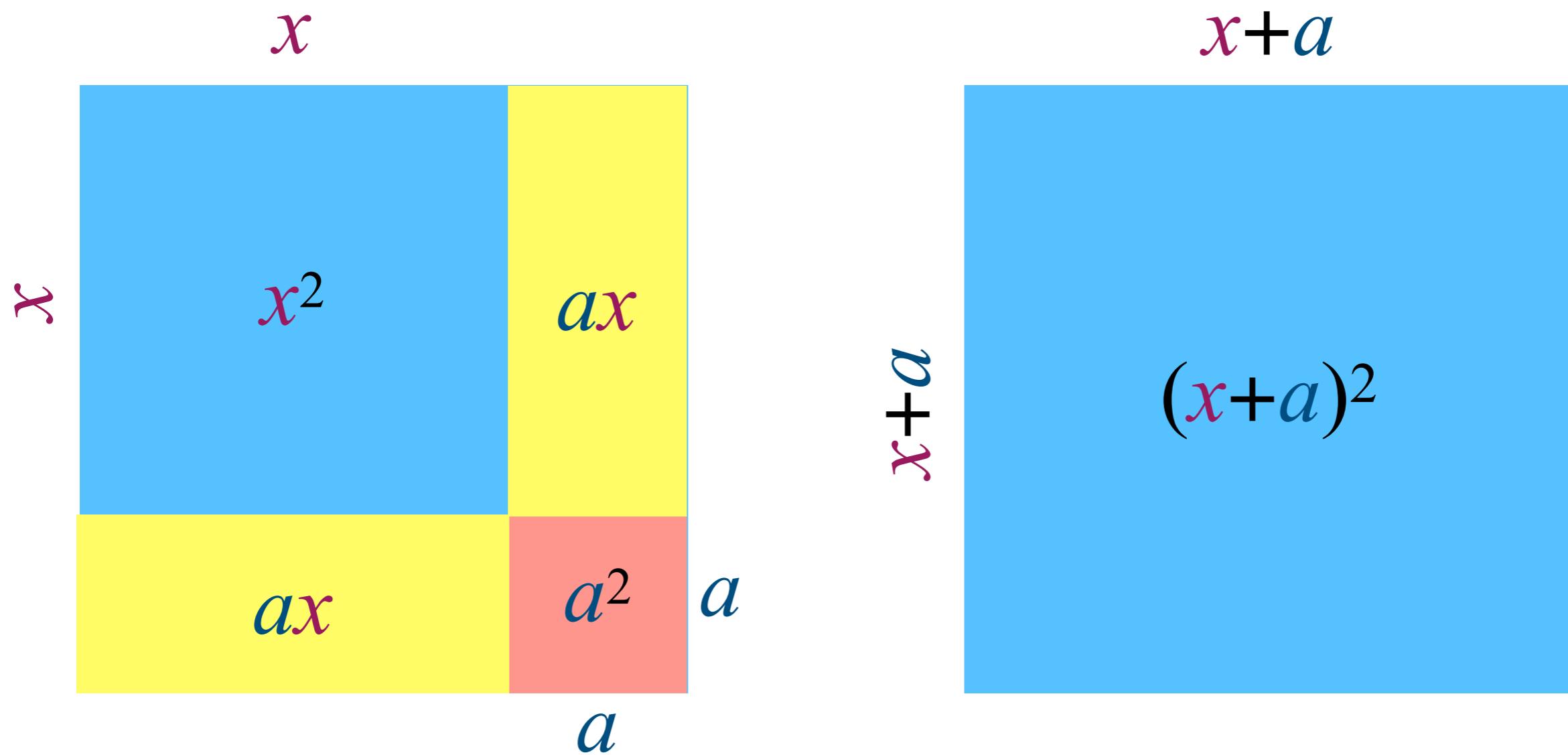
A “solution” is a formula with  $x$  on the left *by itself* and a function involving *only*  $a, b$  &  $c$  on the right.

**But first, a little  
practice...**

# Geometrical Tricks with AREAS:

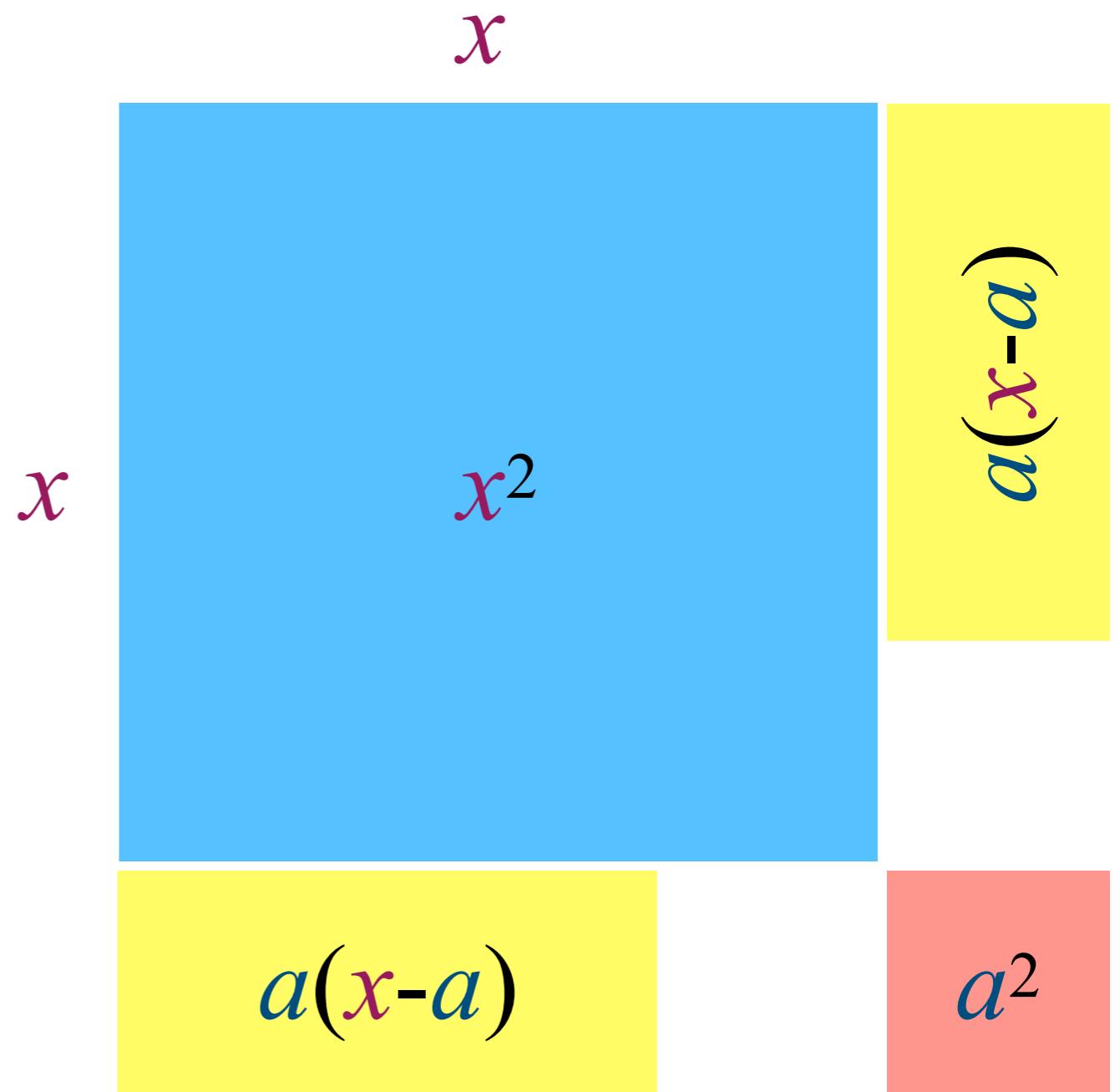
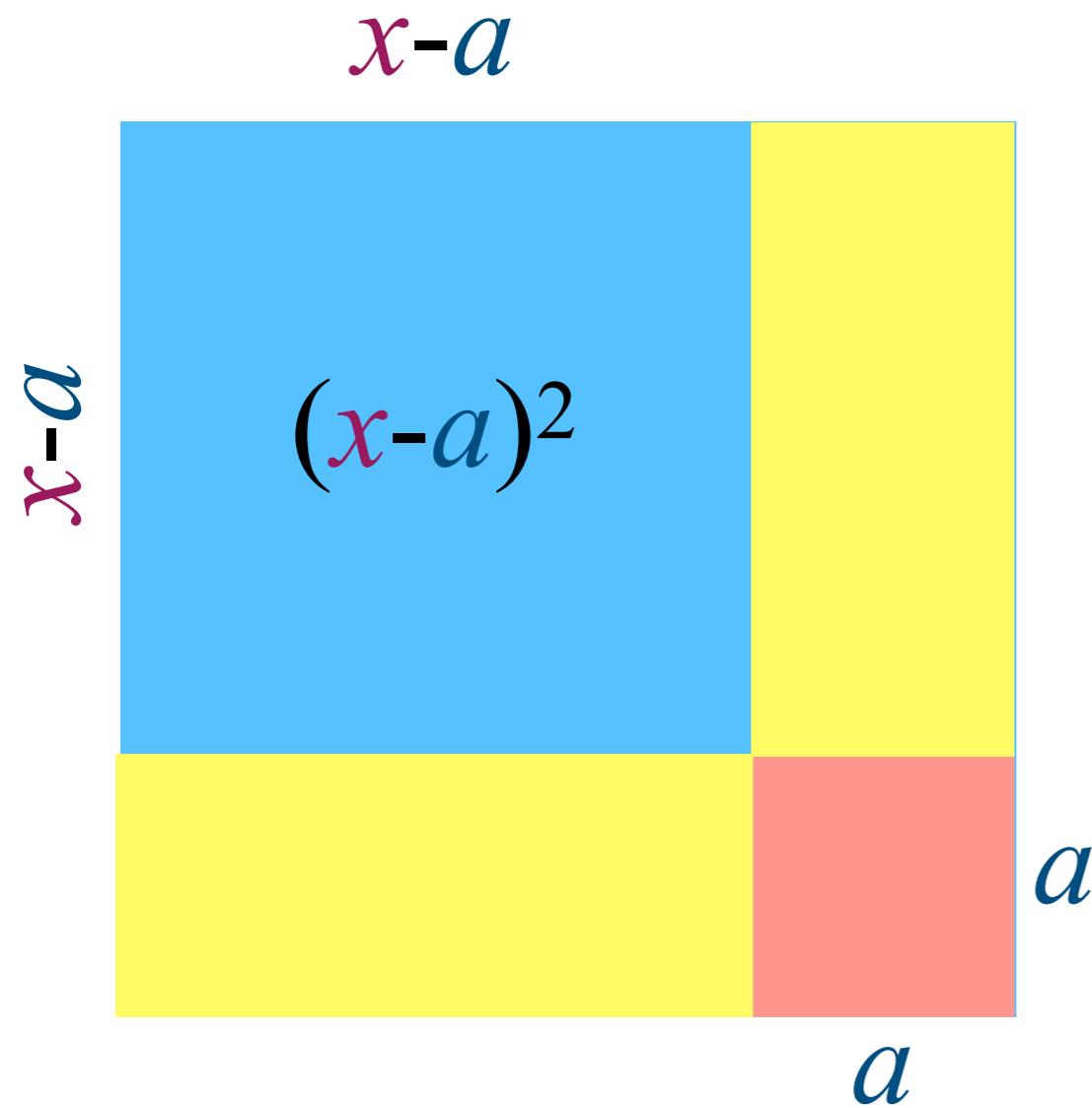


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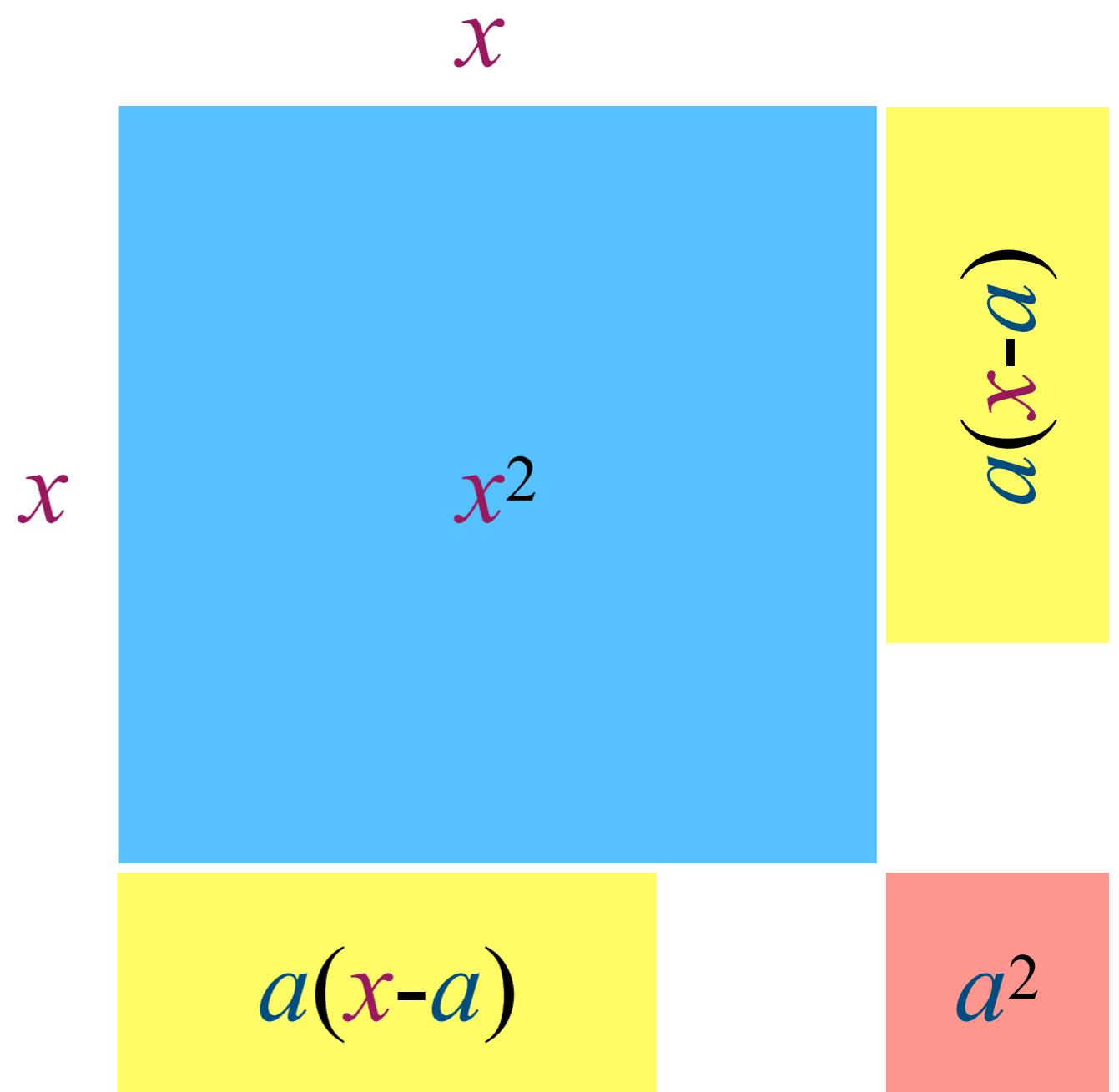
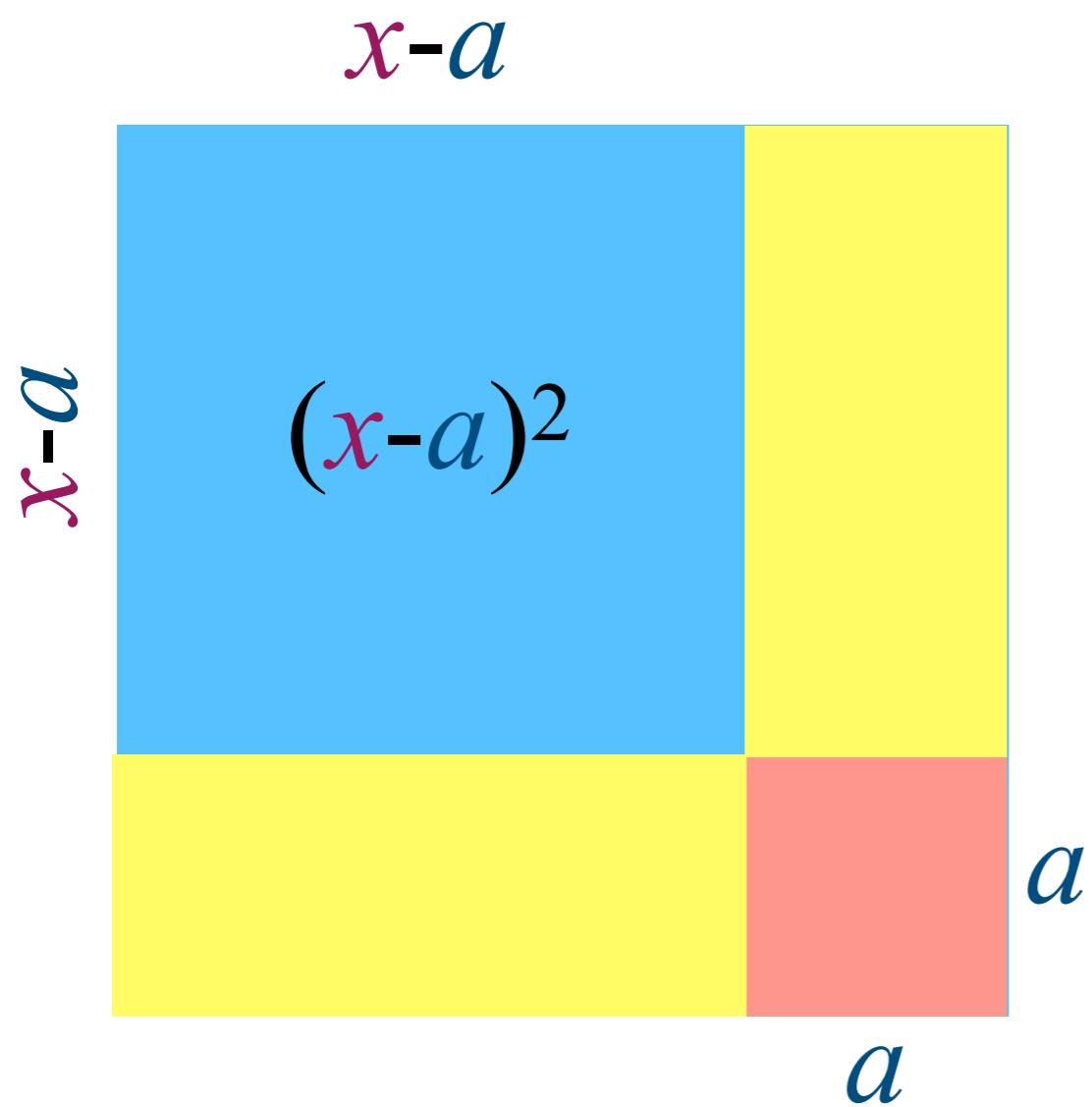


$$(x+a)^2 = x^2 + 2ax + a^2$$

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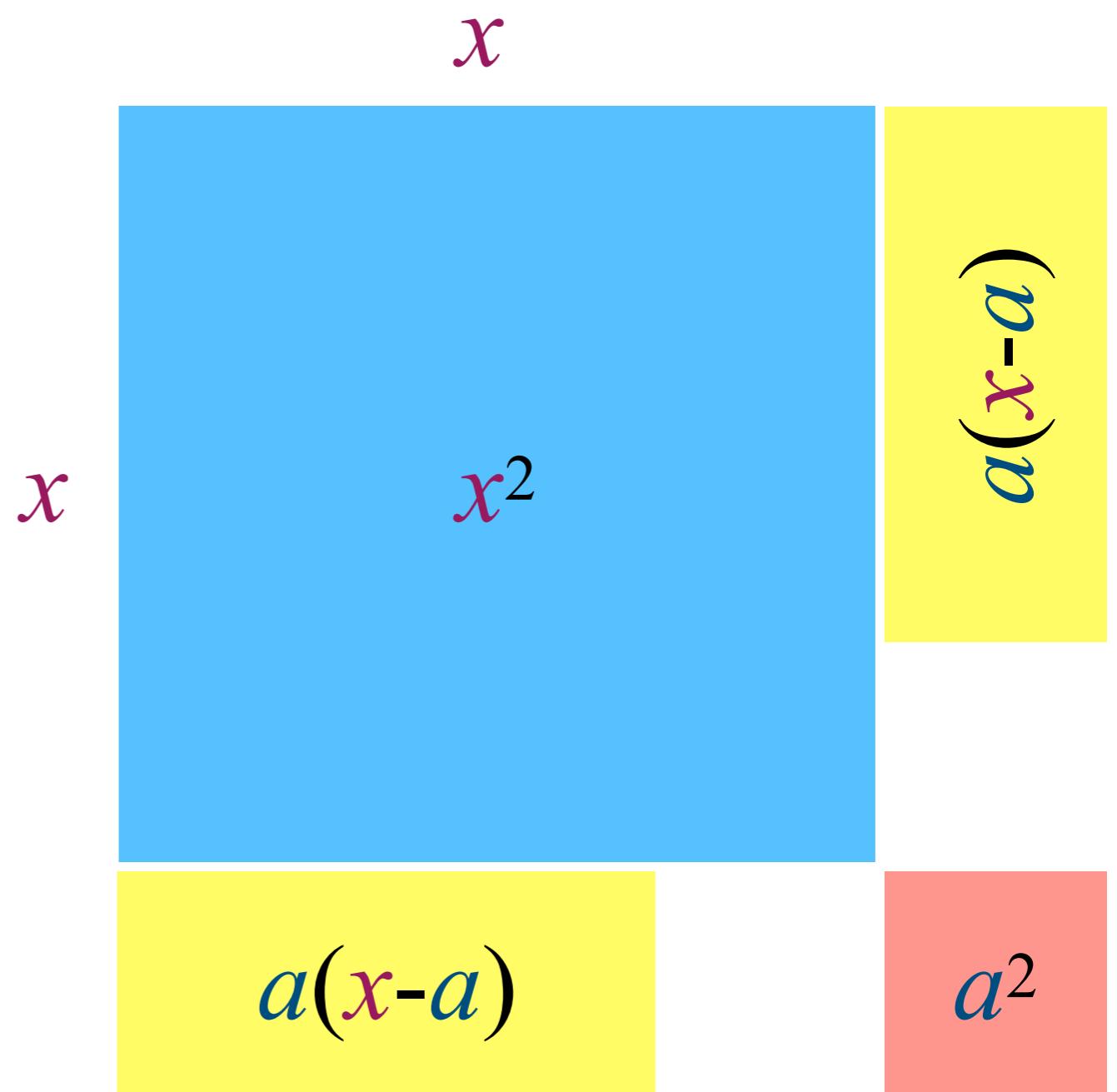
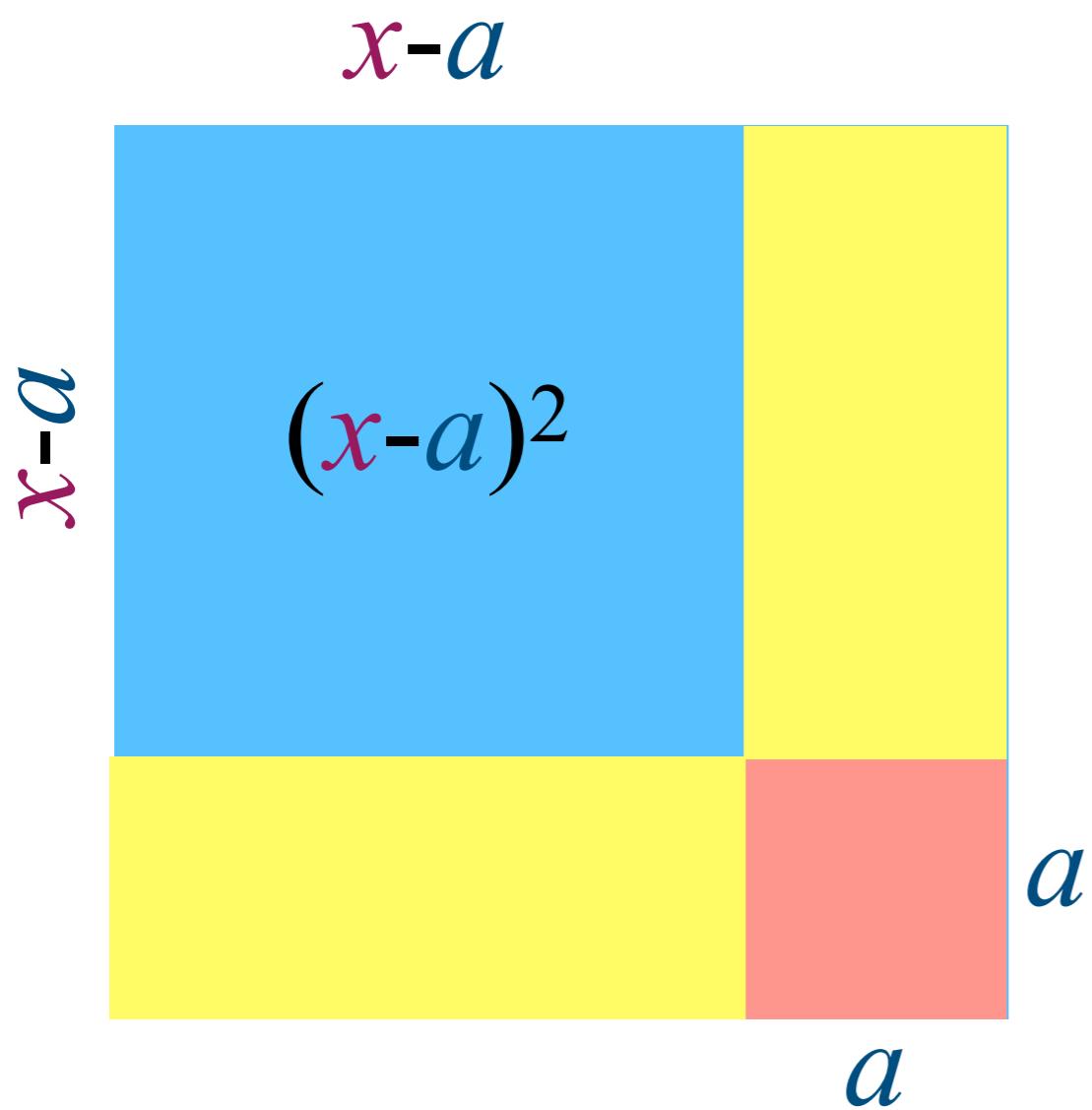


# Geometrical Trick with AREAS:



$$(x-a)^2 = x^2 - 2a(x-a) - a^2$$

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$$(x-a)^2 = x^2 - 2a(x-a) - a^2 = x^2 - 2ax + a^2$$

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# **Back to the Quadratic Equation:**

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$$\begin{array}{r} ax^2 + bx + c = 0 \\ - (c = c) \\ \hline ax^2 + bx = -c \\ \div (a = a) \\ \hline x^2 + (b/a)x = -c/a \end{array}$$

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Now, what can we add to both sides that will make the left side the *square* of something?



$$x^2 + (\textcolor{blue}{b}/a)x :$$

compare

$$(\textcolor{violet}{x}+\textcolor{blue}{d})^2 = x^2 + 2dx + d^2$$

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$$(\textcolor{violet}{x}+\textcolor{blue}{d})^2 = \textcolor{violet}{x}^2 + 2\textcolor{blue}{d}\textcolor{violet}{x} + d^2$$

or  $(\textcolor{violet}{x}+\textcolor{blue}{d})^2 - d^2 = \textcolor{violet}{x}^2 + 2\textcolor{blue}{d}\textcolor{violet}{x}$

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$$\text{or } (\textcolor{violet}{x}+\textcolor{blue}{d})^2 - d^2 = \textcolor{violet}{x}^2 + 2\textcolor{blue}{d}\textcolor{violet}{x}$$

$$\text{so if } 2d = \textcolor{blue}{b}/a, \text{ or } d = \textcolor{blue}{b}/2a,$$

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so if  $2d = \textcolor{blue}{b}/a$ , or  $d = \textcolor{blue}{b}/2a$ ,

$$\textcolor{violet}{x}^2 + (\textcolor{blue}{b}/a)x = (\textcolor{violet}{x}+\textcolor{blue}{b}/2a)^2 - (\textcolor{blue}{b}/2a)^2$$

$x^2 + (b/a)x :$

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$$(x+d)^2 = x^2 + 2dx + d^2$$

or  $(x+d)^2 - d^2 = x^2 + 2dx$

so if  $2d = b/a$ , or  $d = b/2a$ ,

$$x^2 + (b/a)x = (x+b/2a)^2 - (b/2a)^2$$

and  $x^2 + (b/a)x = -c/a$  becomes

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$$(x+b/2a)^2 - (b/2a)^2 = -c/a$$

or  $(x+b/2a)^2 = (b/2a)^2 - c/a$



$$(\textcolor{violet}{x} + \textcolor{blue}{b}/2\textcolor{blue}{a})^2 = (\textcolor{blue}{b}/2\textcolor{blue}{a})^2 - \textcolor{blue}{c}/\textcolor{blue}{a}$$

Take square root of both sides:

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or

$$\textcolor{violet}{x} = \frac{-\textcolor{blue}{b} \pm \sqrt{\textcolor{blue}{b}^2 - 4\textcolor{blue}{a}\textcolor{blue}{c}}}{2\textcolor{blue}{a}}$$

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The Quadratic Theorem

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$$\textcolor{violet}{x}^2 + 2\textcolor{violet}{x} + 1 = 0$$

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$$\textcolor{violet}{x} = \frac{-2 \pm \sqrt{4 - 4}}{2}$$

# EXAMPLE:

$$x^2 + 2x + 1 = 0$$

$$a = 1, \quad b = 2 \quad \& \quad c = 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{4 - 4}}{2} = -1$$

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$$x^2 + 3x + 2 = 0$$

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$$x = \frac{-3 \pm \sqrt{9 - 8}}{2}$$

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$$x = \frac{-3 \pm \sqrt{9 - 8}}{2} = -1 \text{ or } -2$$

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$$x^2 + x + \frac{1}{2} = 0$$

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$$\textcolor{violet}{x}^2 + \textcolor{violet}{x} + \frac{1}{2} = 0$$

$$\textcolor{teal}{a} = 1, \textcolor{teal}{b} = 1 \quad \& \quad \textcolor{teal}{c} = \frac{1}{2}$$

# EXAMPLE:

$$\textcolor{violet}{x}^2 + \textcolor{violet}{x} + \frac{1}{2} = 0$$

$$a = 1, \textcolor{blue}{b} = 1 \quad \& \quad c = \frac{1}{2}$$

$$x = \frac{-\textcolor{blue}{b} \pm \sqrt{\textcolor{blue}{b}^2 - 4ac}}{2a}$$

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# Some Generalizations:

$$ax^2 + bx + c = 0$$

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If  $b^2 = 4ac$ , there is *only one* root.

If  $b^2 - 4ac$  is *negative*, the roots are **complex**.