THE UNIVERSITY OF BRITISH COLUMBIA

# Physics 108 Assignment # **2:**

# KINETIC THEORY OF GASES

Wed. 12 Jan. 2005 — finish by Wed. 19 Jan.

**1. Quantum Tension in a String**

A single electron is confined to a single-walled carbon nanotube (SWNT) of length $L = 1\ \mu$m but is free to move up and down the length of the SWNT. (Since the SWNT is only about 1.2 nm in diameter, you may think of it as a long string). If this system is cooled to nearly 0 K so that the electron is in its lowest possible energy state (the "ground state"), what is the *tension* in the SWNT "string" due to the electron's confinement?

*Hint:* Use de Broglie's hypothesis ($\lambda = h/p$) and think in terms of *standing waves*. Then use a classical picture of a particle of momentum $p = mv$ bouncing back and forth off the ends of the string. . . .

**2. One-Dimensional Ideal Gas**

Making use of the EQUIPARTITION THEOREM, derive an equation analogous to the familiar 3D IDEAL GAS LAW ($pV = N\tau$) for an ideal gas confined to a **one**-dimensional "box" of length $L$. (Some examples would be $N$ electrons moving freely along a single DNA molecule, a *trans*-polyacetylene chain, a SWNT or a "nanowire" made from GaAs/AlGaAs structures.)

**3. One-Dimensional Maxwellian Speed Distribution**

(*a*) What is the thermal speed distribution $\mathcal{D}(v)$ [in the textbook's notation, $N(v)/N$, as in Eq. (22-14) on p. 503] for an ideal gas confined to a **one**-dimensional "box"? It would be nice if you could find the right leading factors (involving temperature and various constants) to *normalize* the distribution so that

$$\int_0^\infty \mathcal{D}(v)\, dv \;=\; 1 \;,$$

but I am mainly looking for its *dependence on the speed* $v$.

*Hint:* Again, use de Broglie's hypothesis and think of *standing waves*.

(*b*) *Sketch* this distribution for a given temperature and compare its shape with that shown in the Figures on p. 503 of the textbook.

(*c*) What can you say about the *most probable speed* $v_{\rm p}$ in the two different cases?