THE UNIVERSITY OF BRITISH COLUMBIA

Physics 108 Assignment # 9 SOLUTIONS:

INDUCTANCE & CIRCUITS

Wed. 9 Mar. 2005 — finish by Wed. 16 Mar.

1. Solenoid as an *RL* Circuit: A long wire with net resistance $R = 120\ \Omega$ is wound onto a nonmagnetic spindle to make a solenoid whose cross-sectional area is $A = 0.02\ \text{m}^2$ and whose effective length is $\ell = 0.5$ m. (Treat the coil as an ideal, long solenoid.) Using a battery with a 1 MΩ internal resistance, a magnetic field of $B_0 = 0.6$ T has been built up inside the solenoid. At $t = 0$ the battery is shorted out and then disconnected so that the current begins to be dissipated by the coil's resistance R. We find that after 3.6 ms the field in the coil has fallen to 0.1 T.

(*a*) How many joules of energy are stored in the coil at $t = 0$? **ANSWER**: The energy density in a magnetic field is given in general by $\dfrac{U}{V} = \dfrac{1}{2\mu_0}B^2$, so the total stored energy is $U = \dfrac{\ell A}{2\mu_0}B^2$, in this case $U_0 = \dfrac{0.5 \times 0.02 \times (0.6)^2}{2 \times 4\pi \times 10^{-7}}$, or $\boxed{U_0 = 1432.4\ \text{J}}$.

(*b*) How long does it take for the stored energy to fall to half its initial value? **ANSWER**: Since $U \propto B^2$, $U \to \frac{1}{2}U_0$ when $B \to \frac{1}{\sqrt{2}}B_0$ and since $B \propto I$, this occurs when $I \to \frac{1}{\sqrt{2}}I_0$. We know that the current in an RL circuit decays exponentially, so $I(t) = I_0 \exp(-t/\tau)$. We are therefore looking for the time when $I/I_0 = 1/\sqrt{2} = \exp(-t_f/\tau)$, or $t_f = \tau \ln(\sqrt{2})$. We can determine the time constant τ from the given fact that $B/B_0 = 0.1/0.6 = \exp(-3.6 \times 10^{-3}/\tau)$ or $\tau = 3.6 \times 10^{-3}/\ln 6 = 2.01 \times 10^{-3}$ s. This then gives $\boxed{t_f = 0.696 \times 10^{-3}\ \text{s}}$.

(*c*) What is the total number of turns in the coil? **ANSWER**: The time constant is $\tau = L/R$. With $R = 120\ \Omega$ this gives $L = 0.241$ H (Henries). It is also true that a solenoid of this form has $L = \mu_0 n^2 A\ell = \mu_0 N^2 A/\ell$, giving $N^2 = \dfrac{\ell L}{\mu_0 A} = \dfrac{0.5 \times 0.241}{4\pi \times 10^{-7} \times 0.02} = 4.80 \times 10^6$ or $\boxed{N = 2190\ \text{turns}}$.

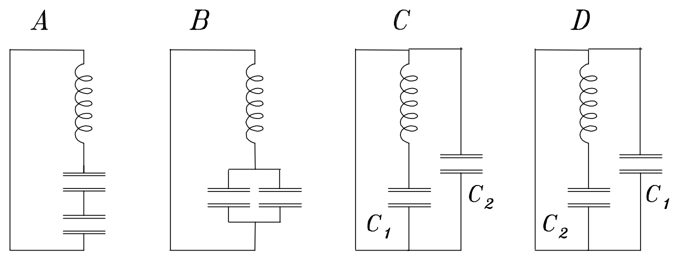
2. *LC* Circuit Time-Dependence: In an LC circuit with $C = 90\ \mu$F the current is given as a function of time by $I = 3.4\cos(1800t + 1.25)$, where t is in seconds and I is in amperes.

(*a*) How soon after $t = 0$ will the current reach its maximum value? **ANSWER**: The phase of the oscillation is $\theta = \omega t + \phi$. By inspection, $\omega = 1800\ \text{s}^{-1}$ and $\phi = 1.25$ rad. Thus $\cos\theta$ will reach its maximum *amplitude* at $\theta = \pi$, for which $t = (\pi - 1.25)/1800$ or $\boxed{t = 1.05\ \text{ms}}$. However, since this gives a maximum *negative* current, one might argue that the *maximum* (positive) current will first occur when $\theta = 2\pi$ or for $t = (2\pi - 1.25)/1800$ or $\boxed{t = 2.80\ \text{ms}}$. Either answer is acceptable.

(*b*) Calculate the inductance. **ANSWER**: Since $\omega = 1/\sqrt{LC} = 1800\ \text{s}^{-1}$, $L \times (90 \times 10^{-6}\ \text{F}) = 1/(1800)^2$, giving $\boxed{L = 3.43 \times 10^{-3}\ \text{H.}}$

(*c*) Find the total energy in the circuit. **ANSWER**: When $i = i_{\text{max}} = 3.4$ A, all the energy is in the inductance L. Later on this gets shared back and forth with the capacitance, but the total energy never changes. Thus $U = \frac{1}{2}Li_{\text{max}}^2 = \frac{1}{2} \times 3.43 \times 10^{-3} \times (3.4)^2$ or $\boxed{U = 0.0198\ \text{J}}$.

3. Build Your Own Circuit: You are given a 12 mH inductor and two capacitors of 7.0 and 3.0 μF capacitance. List all the *resonant frequencies* that can be produced by connecting these circuit elements in various combinations.



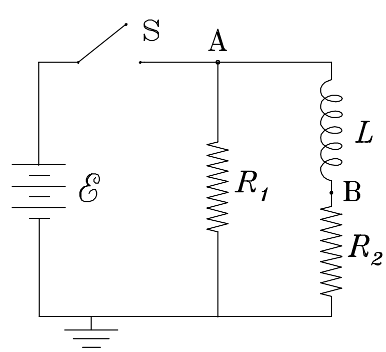
ANSWER: Generally only the loops with *both* an L *and* a C can resonate. Any "external" C is just a "spectator" (consider $\sum \Delta\mathcal{E} = 0$ on the outermost loop in C or D). Thus

$$\omega_A = \left[L\left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1}\right]^{-\frac{1}{2}} = 6299 \text{ s}^{-1}; \quad \omega_C = [LC_1]^{-\frac{1}{2}} = 3450 \text{ s}^{-1}$$

$$\omega_B = [L(C_1+C_2)]^{-\frac{1}{2}} = 2887 \text{ s}^{-1}; \quad \omega_D = [LC_2]^{-\frac{1}{2}} = 5270 \text{ s}^{-1}$$

46.316 0-20.57 0+12

4. *LRR* Circuit Time-Dependence: In the circuit shown, the $\mathcal{E} = 12$ V battery has negligible internal resistance, the inductance of the coil is $L = 0.12$ H and the resistances are $R_1 = 120\ \Omega$ and $R_2 = 70\ \Omega$. The switch **S** is closed for several seconds, then opened. Make a quantitatively labelled graph with an abscissa of *time* (in milliseconds) showing the *potential* of point **A** with respect to ground, just before and then for 10 ms after the opening of the switch. Show also the variation of the potential at point **B** over the same time period.



ANSWER: After S has been closed for a long time, $dI/dt = 0$ and L acts like a plain wire. Then both resistors have the same potential drop as the battery: $\mathcal{E} = I_1R_1 = I_2R_2$, giving $I_1 = 12\text{ V}/120\ \Omega = 0.1$ A and $I_2 = 12\text{ V}/70\ \Omega = 0.1714$ A. Also at that time $\mathcal{E}_A = \mathcal{E}_B = \mathcal{E} = 12$ V. When the switch opens at $t = 0$, the isolated right loop is just an LR circuit with $L = 0.12$ H and $R = R_1 + R_2 = 190\ \Omega$. The current $I_2 = 0.1714$ A flowing through L cannot change suddenly but that through R_1 immediately reverses direction and is thereafter equal to $I_2 = I$. Subsequently $I(t) = [0.1714\text{ A}]e^{-t/\tau}$ where $\tau = L/R = 0.12/190 = 4 \times 10^{-4}$ s. Point A is at a voltage $\mathcal{E}_A = -IR_1$ with respect to ground and point B is at a voltage $\mathcal{E}_B = +IR_2$ with respect to ground. These have initial values $\mathcal{E}_A(0) = 0$ V and $\mathcal{E}_B(0) = 0$ V and both decay exponentially toward zero with time constant τ.