THE UNIVERSITY OF BRITISH COLUMBIA

Physics 108

SOLUTIONS

SECOND MIDTERM - 11 March 2005

TIME: 50 MINUTES

INSTRUCTOR(S): JESS H. BREWER

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1. QUICKIES [10 marks each — 50 total]

(*a*) Show with sketches how to combine

i) two identical capacitors to give an equivalent capacitance half as big as either; [1]

ii) two identical resistors to give an equivalent resistance half as big as either; [2]

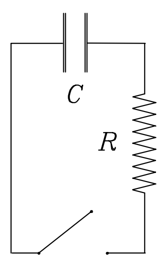
iii) two identical coils to give an equivalent inductance half as big as either; [3]

iv) two identical batteries to give an equivalent voltage half as big as either. [4]

(*b*) Explain why a static magnetic field can't change the kinetic energy of a charged particle. [5]

Describe in *quantitative* detail what happens after the switch is closed at $t = 0$ in each of the following circuits, where $C = 0.1$ F, $R = 10\ \Omega$, $L = 0.01$ H and $\mathcal{E} = 10$ V.
(Graphs are fine as long as the axes are labeled *quantitatively*.)

(*c*) The capacitor is initially charged to $Q = 1$ C: [6]



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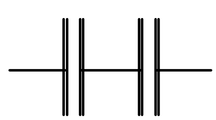
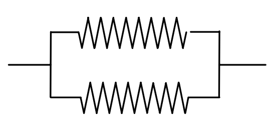
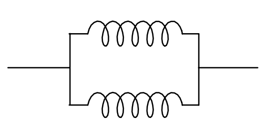
[1]



[2]



[3]

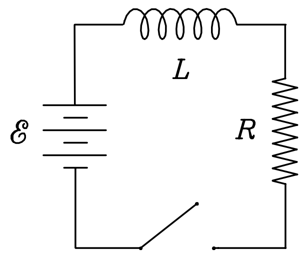


[4]This is impossible, of course. Trust your own conclusions!

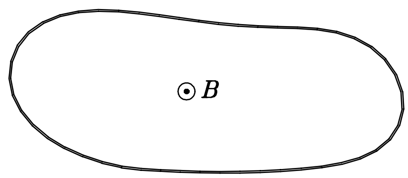
[5]The Lorentz force on a moving charge is $\vec{F} = q\vec{v} \times \vec{B}$, always perpendicular to $\vec{v}$. Thereofre it can only change the *direction* of $\vec{v}$, never its *magnitude*, and so $\frac{1}{2}mv^2$ is constant.

[6]The charge on the capacitor bleeds off through the resistor exponentially: $Q(t) = Q_0 \exp(-t/\tau_{\rm RC})$, where $Q_0 - 1$ C and $\tau_{\rm RC} = RC = 1$ s.

(d) [7]



(e) A loop of limp braided wire lies flat on a frictionless table in an elongated oval shape. Suddenly a uniform magnetic field is turned on normal to the table's surface. What happens to the loop? Why?



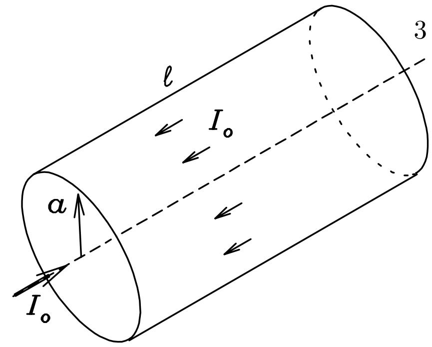
[8]

[7]The current in the inductance builds up from zero, asymptotically approaching the value it would have without the inductance present, namely $I_\infty = \mathcal{E}/R = 1$ A. $I(t) = I_\infty\,[1 - \exp(-t/\tau_{\rm LR})]$, where $\tau_{\rm LR} = L/R = 10^{-3}$ s $= 1$ ms.

[8]The increasing upward flux through the loop induces an EMF around the loop in the direction that will generate its own *downward* field through the loop — *i.e.* clockwise. The *force* on the induced current due to the external field is toward the centre of the loop on both sides, causing it to collapse inward. Since the loop is a flattened oval, this makes it tend to flatten more.

On the other hand, the fields generated by the induced current act repulsively on the opposite side of the loop where the current is going in the opposite direction (opposite currents repel), so we expect the collapse of the loop to stop when the two sides get too close together.

If the loop is *superconducting*, it is not obvious which force "wins" the battle between attraction and repulsion. This is a tantalizing problem and gave you a chance to show off the subtlety of your thinking. Full credit will be given for any answer that makes sense.



2. Ideal Coaxial Cable [50 marks]

An idealized coaxial cable consists of a solid cylindrical wire of length ℓ and radius a coated with a thin layer of insulating paint and a second thin layer of metal (outside the paint, so that it does not make electrical contact with the inner wire). The thickness of the paint and that of the outer conductor are both negligible compared with a, and we shall treat the wire as "long" ($\ell \gg a$) so that "end effects" can be neglected.

A net current I_0 flows down the solid central conductor and back (in the opposite direction) along the thin outer conductor. The current density $\vec{J}$ is uniform over the cross-sectional area of the central conductor and the returning current is uniformly distributed over the surface of the outer conducting shell.

(a) [10 marks] In what *direction* is the vector magnetic field $\vec{B}$ inside and outside the cable? (Indicate on the sketch and/or in words.) [9]

(b) [10 marks] Calculate the magnetic field strength B as a function of r (the distance from the central axis), I_0, a and any fundamental constants, both inside and outside the cable. [10]

(c) [10 marks] Calculate the cable's *inductance* in terms of a and ℓ. [11]

(d) [10 marks] Describe what would happen in the cable if the battery driving the current were suddenly "shorted out" by a superconducting switch. [12]

(e) Now assume that the thin outer conductor has *no* resistance but the solid inner conductor has a resistivity $\rho = 10^{-6}$ Ωm. If $a = 1$ mm and $\ell = 2$ m, what is the *resistance* of the cable? [13]

(f) [5 marks] If $I_0 = 0.5$ A, what is the *electric field* $\vec{E}$ in the inner conductor? [14]

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[9] Inside the wire, $\vec{B}$ loops around the axis in the sense of the right hand rule for the inner current. Outside, $B = 0$.

[10] Outside, $B = 0$ by Ampère's Law because the net current enclosed by a loop around the cable is zero. Inside, Ampère's Law gives $\oint \vec{B} \cdot d\vec{\ell} = 2\pi r B = \mu_o I_{\rm encl} = \mu_o I_0 \left(\frac{\cancel{\pi} r^2}{\cancel{\pi} a^2}\right)$ or $\boxed{B(r < a) = \frac{\mu_o I_0}{2\pi a^2} \cdot r}$.

[11] The magnetic field is completely contained inside the wire; it makes loops around the axis and its strength varies with r. So we make an area element $dA = \ell dr$ consisting of a long strip of width dr at radius r, oriented perpendicular to the field. The magnetic flux through this strip is $d\Phi = B(r) dA = \frac{\mu_o I_0}{2\pi a^2} \ell r dr$ which integrates easily to give $\Phi = \frac{\mu_o I_0}{2\pi a^2} \ell \frac{1}{2} a^2 = \left(\frac{\mu_o \ell}{4\pi}\right) I_0$. The definition of L is $\Phi = LI$, so $\boxed{L = \frac{\mu_o \ell}{4\pi}}$. The only hard part here is visualizing the geometry for the flux integral.

[12] If a battery was required to "drive" the current in the first place, then the cable must have a resistance R, in which case it constitutes an LR circuit and has a time constant $\tau_{\rm LR} = L/R$ for the exponential decay of the current: $I(t) = I_0 \exp(-t/\tau_{\rm LR})$.

[13] The definition of *resistivity* can be expressed as $R = \rho\ell/A$ where $A = \pi a^2$. Thus $R = \frac{10^{-6} \times 2}{\pi \times (10^{-3})^2}$ or $\boxed{R = \frac{2}{\pi} = 0.6366\ \Omega}$.

[14] You can express Ohm's Law either as $\vec{E} = \rho \vec{J}$, in which case $E = \rho \frac{I_0}{\pi a^2}$, or as $V = IR$ where $V = E\ell$, in which case $E = \frac{I_0 R}{\ell} = \frac{0.5 \times 2/\pi}{2}$ or $\boxed{E = \frac{0.5}{\pi} = 0.159 \text{ V/m}}$.