The University of British Columbia

Physics 122 Assignment # 4:

GAUSS' LAW

Fri. 25 Jan. 2002 — finish by Fri. 1 Feb.

1. GAUSS' LAW FOR A SHEET OF CHARGE: Imagine an infinite plane sheet of electric charge with $dQ/dA = \sigma$ units of charge per unit surface area. *Using your own words and drawings,*[1]

(a) Using only (i) the SUPERPOSITION PRINCIPLE for the total electric field due to an assembly of electric charges, plus (ii) simple SYMMETRY arguments, deduce the *direction* of the electric field due to the plane of charge.

(b) Using the preceding result plus the general ideas of Gauss' Law and simple geometry, deduce the dependence of the *magnitude* E of the electric field upon x, the *perpendicular distance* away from the plane.

(c) Show that your result agrees with the x-dependence of the electric field on axis due to a uniform *disc* of charge when $x \ll R$, the radius of the disc.

2. ATOMS AS SPHERES OF CHARGE: In Rutherford's work on α particle scattering from atomic nuclei, he regarded the atom as having a pointlike positive charge of $+Ze$ at its centre, surrounded by a spherical volume of radius R filled with a uniform charge density that makes up a total charge $-Ze$. In this simple model, calculate the electric field strength E and the electric potential ϕ as functions of radius r and various constants. *Plot* your results for $0 < r \leq 2R$. (Choose $\phi \underset{r\to\infty}{\longrightarrow} 0$.)

3. FIELD WITHIN A UNIFORM CHARGE DISTRIBUTION: The textbook shows how to use Gauss' Law to derive the radial (r) dependence of the electric field $E(r > R)$ *outside* charge distributions of spherical, cylindrical or planar symmetry, where R is the distance the charge distribution extends from the centre of symmetry — the radius of a charged *sphere* or *cylinder*, or half the thickness of an infinite *slab* of charge, respectively. Use similar arguments to show that, for each of these cases (a sphere, cylinder or a slab of uniform charge density), the electric field $E(r < R)$ *inside* the charge distribution is given in terms of the field $E(R)$ *at the boundary* of the charge distribution by

$$E(r < R) = \left(\frac{r}{R}\right) E(R).$$

[1] The full derivation can be found in any textbook; however, you will not really *own* Gauss' Law until you have expressed and used it *your own way*. This is true for almost everything, but Gauss' Law is so important (and so *simple!*) that you should not miss this chance to make it yours.