The University of British Columbia

Physics 401 Assignment # 4: POTENTIALS, GAUGES and RELATIVITY

Wed. 25 Jan. 2006 — finish by Wed. 1 Feb.

Please review Section 10.1 and Ch. 12.

1. (p. 420, Problem **10.3**) — **GIVEN** V **&** $\vec{\boldsymbol{A}}$ **. . .** Find the $\vec{\boldsymbol{E}}$, $\vec{\boldsymbol{B}}$, ρ & $\vec{\boldsymbol{J}}$ corresponding to

$$V(\vec{\boldsymbol{r}}, t) = 0, \qquad \vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}, t) = -\frac{1}{4\pi\epsilon_0} \frac{qt}{r^2} \,\hat{\boldsymbol{r}} \;.$$

2. **POINT CHARGE:**

- (a) Find the $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$ fields corresponding to a stationary point charge q situated at the origin.
- (b) State the charge and current distributions of this situation.
- (c) What are the electric and magnetic potentials?
- (d) Is there any relation between this situation and that described in Problem **10.3**?

3. (p. 420, Problem **10.5**) — **GAUGE TRANSFORMATION:** Use the gauge function

$$\lambda = -\frac{1}{4\pi\epsilon_0} \left(\frac{q\,t}{r}\right)$$

to transform the potentials in Problem **10.3**, and comment on the result.

4. **WHICH GAUGE?**

- (a) In Problem **10.3** above, are the potentials in the Coulomb gauge, the Lorentz gauge, both, or neither?
- (b) In Problem 2 above, are the potentials in the Coulomb gauge, the Lorentz gauge, both, or neither?

5. **NATURAL UNITS:** Since c is now a *defined* quantity that keeps appearing in confusing places in our notation for 4-vectors *etc.*, and since *nanoseconds* (ns) are perfectly handy units for *distance*, it seems silly to not just measure time and distance in the same units (seconds) and set $c = 1$. While we're at it, why not set the ubiquitous constant in quantum mechanics to unity as well ($\hbar = 1$) so that all angular momenta are unitless and (because $E = \hbar\omega$) energies are measured in s^{-1}?

- (a) In what units would we then measure velocities, momenta, masses, forces and accelerations?
- (b) Suppose we set the Coulomb force constant $k_E \equiv \frac{1}{4\pi\epsilon_0} = 1$ as well. In what units would we then measure charge, electric field, magnetic field, and potentials V and $\vec{\boldsymbol{A}}$?
- (c) Write out Maxwell's equations in this system of units. (*Hint:* We must have $\epsilon_0\,\mu_0 = 1$.)[1]

6. **4-POTENTIAL:** In Eq. (12.131) on p. 541, Griffiths states that, "As you might guess, V and $\vec{\boldsymbol{A}}$ together constitute a 4-vector: $A^\mu = (V/c, A_x, A_y, A_z)$." This is a very strong statement with profound consequences. You can't just take any 3-vector and combine it with a convenient scalar in the same units to make a true 4-vector! *Explain why* we should believe this about A^μ, and list any essential *conditions* that must be met for it to be true.

[1] You are allowed to consult the literature on this, or even `http://en.wikipedia.org/wiki/Planck_units`, but please explain your own reasoning!)