The University of British Columbia

Physics 401 Assignment # 6: ELECTROMAGNETIC WAVES

Wed. 8 Feb. 2006 — finish by Wed. 22 Feb.

1. CMBR: Most of the electromagnetic energy in the universe is in the cosmic microwave background radiation (CMBR), sometimes referred to as the 3° Kelvin background. Penzias and Wilson discovered the CMBR in 1965 using a radio telescope, and subsequently received the Nobel Prize for this discovery. This background radiation has wavelength $\lambda \sim 1.1$ mm. The energy density of the CMBR is about 4.0×10^{-14} J/m^3. What is the *rms* electric field strength of the CMBR?

2. STANDING WAVES: Consider standing electromagnetic waves:

$$\vec{\boldsymbol{E}} = E_0 (\sin kz \ \sin \omega t)\, \hat{\boldsymbol{x}} \qquad \text{with} \qquad \vec{\boldsymbol{B}} = B_0 (\cos kz \ \cos \omega t)\, \hat{\boldsymbol{y}} \ .$$

(*a*) Show that these satisfy the wave equation (9.2).

(*b*) Show that we must also have $c = \omega/k$ and $E_0 = cB_0$.

(*c*) Show that the time-averaged power flow across *any* area will be zero.

(*d*) Show that the Poynting vector will also be zero, *i.e.* there is no net energy flow.

3. (p. 386, Problem **9.14**) — **REFLECTED & TRANSMITTED POLARIZATION:** In Eqs. (9.76) and (9.77) it was tacitly assumed that the reflected and transmitted waves have the same *polarization* as the incident wave, namely along the $\hat{\boldsymbol{x}}$ direction. Prove that this *must* be so. [*Hint:* Let the polarization vectors of the reflected and transmitted waves be

$$\hat{\boldsymbol{n}}_T = \cos\theta_T \hat{\boldsymbol{x}} + \sin\theta_T \hat{\boldsymbol{y}} \qquad \text{and} \qquad \hat{\boldsymbol{n}}_R = \cos\theta_R \hat{\boldsymbol{x}} + \sin\theta_R \hat{\boldsymbol{y}}$$

and prove from the boundary conditions that $\theta_T = \theta_R = 0$.]

4. (p. 392, Problem **9.15**) — **COMPLEX ALGEBRA EXERCISE:** Suppose that we have six nonzero constants A, B, C, a, b, c such than $Ae^{iax} + Be^{ibx} = Ce^{icx}$ for all x. Prove that $a = b = c$ and $A + B = C$.

5. (p. 392, Problem **9.17**) — **DIAMOND:** The index of refraction of diamond is 2.42. Construct the graph analogous to Figure 9.16 for the air/diamond interface. (Assume $\mu_1 = \mu_2 = \mu_0$.) In particular, calculate

(*a*) the amplitudes at normal incidence;

(*b*) Brewster's angle; and

(*c*) the "crossover" angle at which the reflected and transmitted amplitudes are equal.

6. PLANE WAVE STRESS TENSOR: Find all the elements of the Maxwell stress tensor of a monochromatic plane wave traveling in the z-direction, polarized in the x-direction:

$$\vec{\boldsymbol{E}}(z,t) = E_0 \cos(kz - \omega t + \delta)\hat{\boldsymbol{x}}$$

$$\vec{\boldsymbol{B}}(z,t) = \frac{E_0}{c} \cos(kz - \omega t + \delta)\hat{\boldsymbol{y}}$$

In what direction does this EM wave transport momentum? Does this agree with the form of the Maxwell stress tensor you just deduced?

7. (p. 412, Problem **9.33**) — **SPHERICAL WAVES:** Suppose

$$\vec{\boldsymbol{E}}(r,\theta,\phi,t) = A\frac{\sin\theta}{r}\left[\cos(kr-\omega t) - \left(\frac{1}{kr}\right)\sin(kr-\omega t)\right]\hat{\boldsymbol{\phi}} \qquad \text{with} \qquad c = \frac{\omega}{k}\ ,$$

as usual. [This is, incidentally, the simplest possible **spherical wave**. For notational convenience, let $(kr - \omega t) \equiv u$ in your calculations.]

(*a*) Show that $\vec{\boldsymbol{E}}$ obeys all four of Maxwell's equations, in vacuum, and find the associated magnetic field.

(*b*) Calculate the Poynting vector. Average $\vec{\boldsymbol{S}}$ over a full cycle to get the intensity vector $\vec{\boldsymbol{I}}$. Does $\vec{\boldsymbol{I}}$ point in the expected direction? Does it fall off like r^{-2}, as it should?

(*c*) Integrate $\vec{\boldsymbol{I}} \cdot d\vec{\boldsymbol{a}}$ over a spherical surface to determine the total power radiated. [You should get $P = 4\pi A^2/3\mu_0 c$.]