The University of British Columbia

Physics 401 Assignment # 8:
GUIDED WAVES

Wed. 1 Mar. 2006 — finish by Wed. 8 Mar.

1. (p. 405, Problem **9.25**) — **Group Velocity:**[1] Assuming negligible damping ($\gamma_j \approx 0$), calculate the group velocity ($v_g \equiv d\omega/dk$) of the waves described by Eqs. (9.166) and (9.169):

$$\tilde{\boldsymbol{E}}(z,t) = \tilde{E}_0 \, e^{-\kappa z} \, e^{i(kz-\omega t)} \qquad \text{and} \qquad \tilde{k} \simeq \frac{\omega}{c}\left[1 + \frac{Nq^2}{2m\epsilon_0} \sum_j \frac{f_j}{\omega_j^2 - \omega^2 - i\gamma_j\omega}\right] .$$

Show that $v_g < c$, even when $v > c$.

2. (p. 411, Problem **9.27**) — **No TE$_{00}$:** Show that the mode TE$_{00}$ cannot occur in a rectangular wave guide. [*Hint:* In this case $\omega/c = k$, so Eqs. (9.180) are indeterminate, and you must go back to Eqs. (9.179). Show that B_z is a constant, and hence — applying Faraday's law in integral form to a cross section — that $B_z = 0$, so this would be a TEM mode.]

3. (p. 411, Problem **9.28**) — **TE Modes:**[2] Consider a rectangular wave guide with dimensions 2.28 cm × 1.01 cm. What TE modes will propagate in this waveguide, if the driving frequency is 1.70×10^{10} Hz? Suppose you wanted to excite only *one* TE mode; what range of frequencies could you use? What are the corresponding wavelengths (in open space)?

4. (p. 411, Problem **9.30**) — **TM Modes:**[3] Work out the theory of TM modes for a rectangular wave guide. In particular, find the longitudinal electric field, the cutoff frequencies, and the wave and group velocities. Find the ratio of the lowest TM cutoff frequency to the lowest TE cutoff frequency, for a given wave guide. [*Caution:* What is the lowest TM mode?]

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[1] You'll get that the group velocity, $v_g = \dfrac{c}{1 + \frac{q^2 N}{2m\epsilon_0} \sum_j \frac{(\omega_j^2+\omega^2) f_j}{\left(\omega_j^2 - \omega^2\right)^2}}$

[2] You'll find that only 4 **TE** modes will propagate.

[3] You will deduce that $E_z = E_0 \sin\frac{m\pi x}{a} \sin\frac{n\pi x}{b}$ with m, n integers $(1, 2, 3, 4, \cdots)$ and you will find that the cutoff frequency is $\omega_{mn} = c\pi\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$, the same as for TE modes.