UBC Physics 401 Lecture:

COAX CABLES

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http://musr.physics.ubc.ca/p401/pdf/CoaxCablesPlus.pdf

Coaxial Cylinders

Griffiths goes to some length to explain why TEM modes (with $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$ perpendicular to $\vec{\boldsymbol{K}} = k\hat{\boldsymbol{z}}$, *i.e.* a normal transverse wave propagating straight down the guide) **cannot** exist in a **hollow** waveguide because the empty region is surrounded by an equipotential.

Then he states that this restriction is lifted if there is a central conductor. (In fact it is lifted any time there are **two** parallel conductors involved, as for a simple pair of wires or two parallel *strips* of conductor.) From there he *derives* the equivalence of the differential equations describing the transverse field components to MAXWELL'S EQUATIONS IN TWO DIMENSIONS.

I'd like to take a simpler approach: just take our known results for *static* EM fields and impose a wavelike variation along the z axis perpendicular to both.

We have solved for the electric field between two coaxial cylinders with opposite charges per unit length λ *and* for the magnetic field between two coaxial cylinders carrying equal and opposite currents I:

$$\vec{\boldsymbol{E}}(r) = \frac{\lambda}{2\pi\epsilon}\left(\frac{\hat{\boldsymbol{r}}}{r}\right) \qquad \text{and} \qquad \vec{\boldsymbol{B}}(r) = \frac{\mu I}{2\pi}\left(\frac{\hat{\boldsymbol{\theta}}}{r}\right) \tag{1}$$

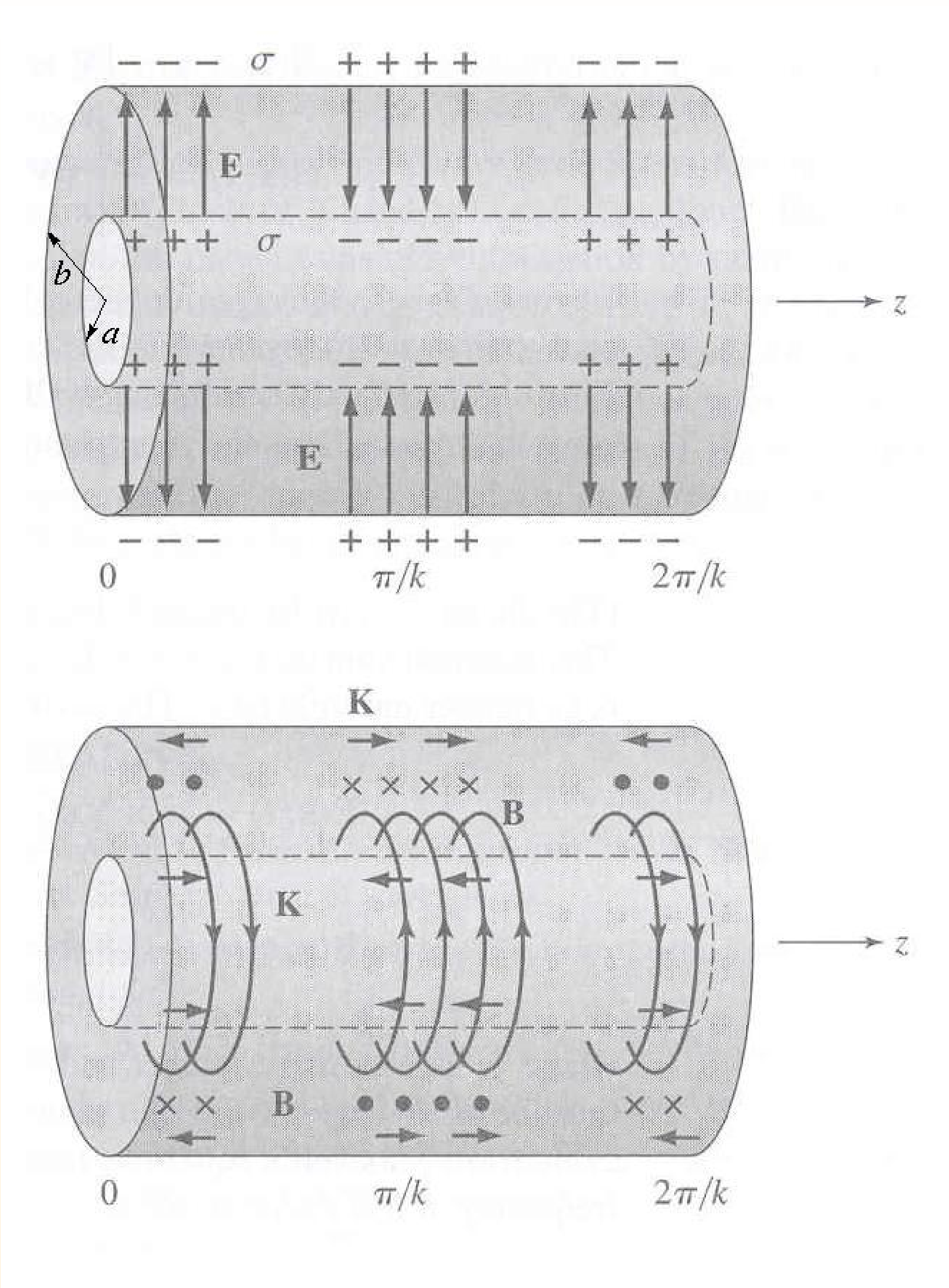
where we have allowed for the possibility of a linear dielectric and/or magnetic medium between the inner and outer conductors.

If we let

$$\lambda = \lambda(z,t) = \lambda_0 e^{i(kz-\omega t)} \qquad \text{and} \qquad I = I(z,t) = I_0 e^{i(kz-\omega t)} \;, \tag{2}$$

the result can easily be shown to satisfy MAXWELL'S EQUATIONS and all the required boundary conditions.

This falls into the "Try the simplest case and see if it works!" category, which we all know and love.



At a given time t it looks like this.

You have often calculated the potential difference $V = -\int_a^b \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{r}}$ between the inner and outer conductors at a given z position in terms of the instantaneous charge per unit length at that z:

$$V(z,t) = \frac{\lambda(z,t)}{2\pi\epsilon_0} \ln\left(\frac{b}{a}\right) . \qquad (3)$$

This voltage depends on z and t as

$$V(z,t) = V_0 \, e^{i(kz-\omega t)} . \qquad (4)$$

Of course, we can't measure λ directly with a voltmeter; but we *can* invert Eq. (3):

$$\lambda(z,t) = \left[\frac{2\pi\epsilon_0}{\ln(b/a)}\right] V(z,t)\,, \tag{5}$$

so the corresponding electric field is

$$\vec{\boldsymbol{E}}(r,z,t) = \frac{V(z,t)}{\ln(b/a)}\left(\frac{\hat{\boldsymbol{r}}}{r}\right)\,. \tag{6}$$

As usual, $E = vB$ where $v = (\epsilon\mu)^{-1/2}$ is the speed of light in the medium between the inner and outer conductors (which is actually usually filled with an insulator for practical reasons). Thus the magnetic field is

$$\vec{\boldsymbol{B}}(r,z,t) = \frac{V(z,t)}{v\ln(b/a)}\left(\frac{\hat{\boldsymbol{\theta}}}{r}\right)\,. \tag{7}$$

Impedance

The (z, t) dependence of the fields is entirely due to

$$V(z,t) = V_0 \, e^{i(kz - \omega t)} \tag{8}$$

and the current is [comparing Eqs. (1) with Eqs. (6) and (7)]

$$I(z,t) = \left[\frac{2\pi V_0}{\mu v \ln(b/a)} \right] e^{i(kz - \omega t)} \, . \tag{9}$$

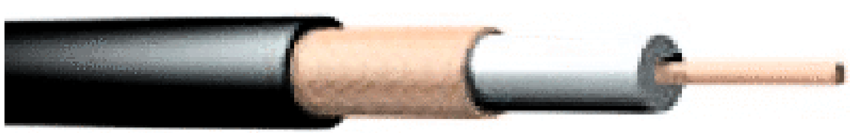
Since $\mu v = \sqrt{\mu/\epsilon}$, the **impedance** $Z \equiv V/I$ is given by

$$Z = \frac{\mu v}{2\pi} \ln\left(\frac{b}{a}\right) = \frac{\ln(b/a)}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \, . \tag{10}$$

For $\mu = \mu_0$ and $\epsilon = \epsilon_0$ the numerical value of Z is (59.96 $\Omega) \cdot \ln(b/a)$. A typical coax cable has $a \approx 0.4$ mm and $b \approx 1.6$ mm, so $\ln(b/a) \approx 1.4$, $\mu \approx \mu_0$ and $\epsilon \approx 3\epsilon_0$ to make $Z \approx 50 \ \Omega$.

RG58 Cables

These are the "workhorse" cables used in any experiments where sharp timing signals must be transported without dispersion over distances of up to ~ 100 m. They are carefully designed to have an impedance of exactly 50 Ω.



RG58 50 Ohm coax cable with 7/0.3mm centre conductor, 64 strand braided screen and a solid polyethylene insulator.