Experiment 6

Magnetic Resonance Imaging in 1 Dimension

Disclaimer: It is absolutely essential that the student read and understand the previous experiment on nuclear magnetic resonance before proceeding with magnetic resonance imaging. The physics of resonance and relaxation of free protons in a DC magnetic field will be assumed knowledge in the following discussion.

6.1 Frequency Encoding by NMR

Medical diagnostic imaging is a branch of applied physics that has experienced explosive growth in the past two decades. The growth has been fueled by many recent developments in electronics and of course very lucrative financial incentives. However, it must be noted that much of the success of medical diagnostic imaging is built upon a solid foundation of fundamental physics that has been understood for almost half a century. Magnetic resonance imaging (MRI) is one such example, as it is a byproduct of research in nuclear magnetic resonance.

To begin, recall that the fundamental resonance condition for free protons in a static homogeneous magnetic field $\vec{B_o}\hat{z}$ is

$$\omega_o = \gamma|\vec{B_o}| \tag{6.1}$$

where $\gamma = 2.675 \times 10^8$ rad/sec T is the gyromagnetic ratio for protons. Application of a radio frequency (rf) pulse with angular frequency ω_o can coherently move the spin population into an excited state. The net magnetization $\vec{M}$ of that state can be measured as a function of time as it decays back to the ground state while precessing at the Larmor frequency in the $\widetilde{xy}$ plane.

$$|\vec{M}(t)| \propto e^{-t/T_2^*} \tag{6.2}$$

This signal is referred to as a *free induction decay* (FID). The relaxation time T_2^* captures the physics of all interactions which result in a loss of coherence within the spin system.

Now consider an extended inhomogeneous sample in one dimension with the spin density at the point z defined as $\rho(z)$.

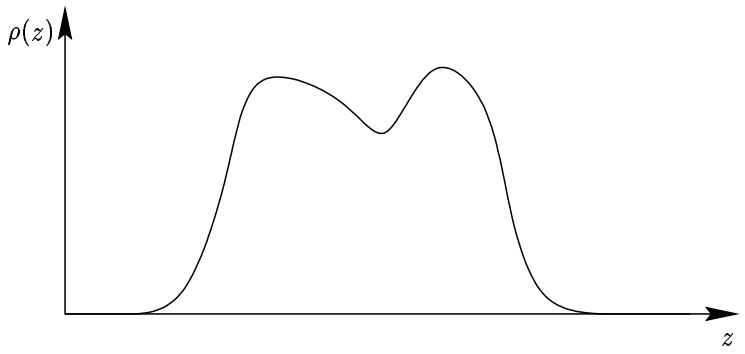

Figure 6.1: The density of an extended inhomogeneous sample in one dimension.

If the sample is subjected to a static uniform magnetic field $\vec{B}_o$, then all spins in the sample will resonate at the same frequency and each point z in the sample will contribute to $\vec{M}$ an amount proportional to $\rho(z)$. However, if the sample is subjected to a static field of the form

$$\vec{B} = (B_o + Gz)\hat{z} \tag{6.3}$$

where B_o and G are constants, then the resonance condition will vary linearly across the sample.

$$\omega_{resonance} = \gamma(B_o + Gz) \tag{6.4}$$

The similarity of Eqns. 6.3 and 6.4 is very suggestive; if one can instantaneously excite the entire sample with a single rf pulse, then the ensuing relaxation of $\vec{M}(t)$ will be a composite of contributions from all parts of the sample precessing at different frequencies, and the magnitude of the contribution at each frequency will depend upon the local spin density at a single point in the sample.

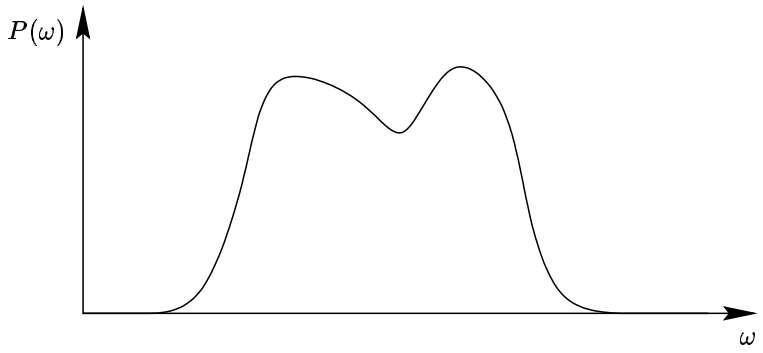

Figure 6.2: The power spectrum obtained from a sample subjected to a static field of the form Eqn. 6.3 and a broadband rf signal. The mapping $\omega_{resonance} \mapsto z$ shows that $\rho(z) \propto \text{Power}(\omega_{resonance}(z))$.

Therefore, the *power spectrum* (magnitude squared of the Fourier transform) of the precession signal $\vec{M}(t)$ will be a 1:1 mapping of the spatial distribution of spins in the sample. Imaging by this means is referred to as *frequency encoding* - the condition for NMR allows one to convert position into frequency and spin density into power. By measuring power as a function of frequency, one then obtains density as function of position.

6.2 Signal Processing Considerations

Now that the essential physics has been explained, one must deal with the practical issues of generating an image. In particular, two key criteria must be considered:

- The entire sample must be visible within the *field of view* (FOV). Any apparatus will only be capable of measuring frequencies over a finite bandwidth. Therefore, the gradient G and static field B_o must be selected such that the resonance frequency on one extreme of the sample will be no less than the minimum observable frequency ω_{min}, and the resonance frequency on the other extreme will be no more than the maximum observable frequency ω_{max}.
- The resolution of the apparatus must meet the specifications for the task at hand. In this particular experiment, this means applying a sufficiently strong gradient G such that the signals from spatially separated spin populations do not overlap in the power spectrum.

These criteria will be the guiding principles upon which experimental parameters will be based.

To begin, one must first determine how to extract frequency information using the NMR spectrometer in the Physics 409 laboratory *without* a gradient field. During an NMR experiment one typically examines

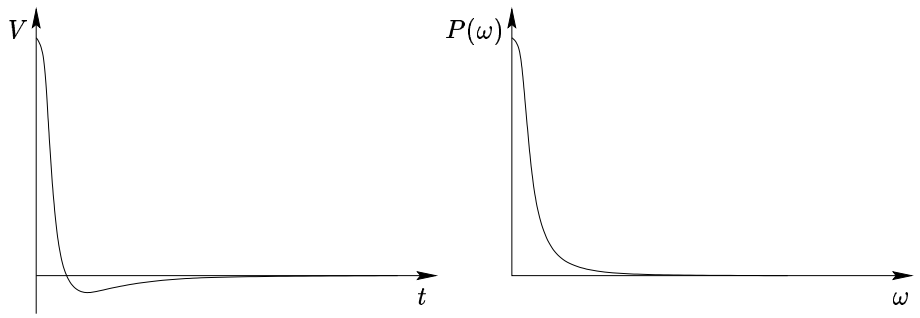

Figure 6.3: An NMR signal from the spectrometer tuned on resonance. The power spectrum of this signal yields no frequency information because the mixer moves the resonance to $\omega = 0$.

the output of the mixer, and that signal will oscillate at the difference frequency between the applied rf pulse and the resonance frequency of the spin system, $\omega_\delta = |\omega_o - \omega_{rf}|$. For convenience, one often tunes the system until $\omega_o = \omega_{rf}$, and then the output of the mixer will be zero.

However, this is not an appropriate strategy for MRI because having the system on resonance effectively nulls any frequency information. Instead, consider the FID as measured by a system slightly off resonance:

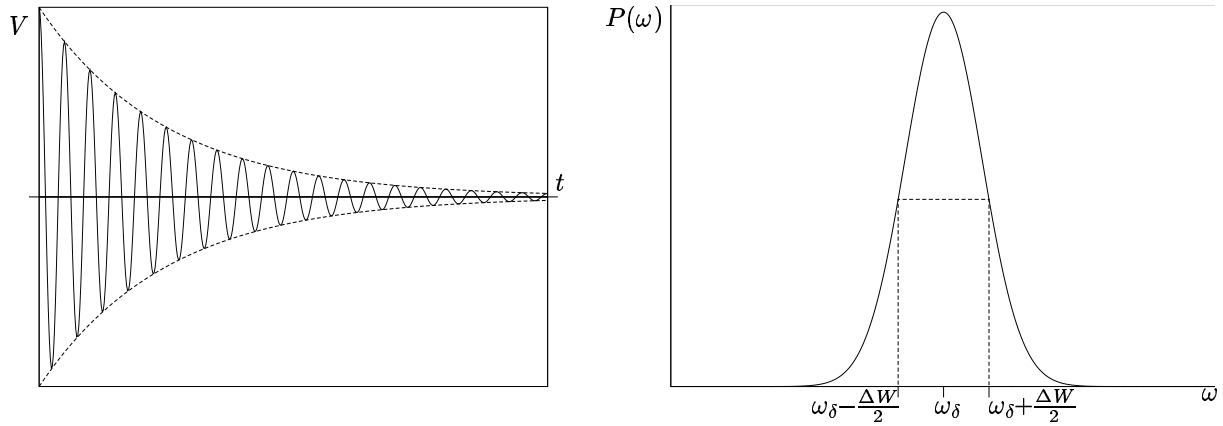

Figure 6.4: An NMR signal from the spectrometer tuned slightly off resonance. The power spectrum of this signal now has a fully resolved peak at the beat frequency of the mixer signal, ω_δ.

By tuning slightly off resonance, one can see the entire spectrum associated with a FID signal - it is typically Gaussian in shape with a characteristic width ΔW because of variations in the static field B_o and the random nature of spin-spin interactions that change the resonance frequency on a local level.

Now consider a sample which has nonzero spin density at only two points z_1 and z_2.

$$\rho(z) = \rho_o\{\delta(z - z_1) + \delta(z - z_2)\}$$

With only the field B_o present, each spin population will resonate at the same frequency and Fig. 6.4 will be very representative of the measured FID. However, if one supplies a gradient G then the spatially separated spin populations will resonate at frequencies ω_1 and ω_2, respectively, as determined from Eqn. 6.4.

$$P(\omega) = P_o\{\delta(\omega - \omega_1) + \delta(\omega - \omega_2)\}$$

Furthermore, the spectral features will be broadened by the same mechanisms responsible for the Gaussian in Fig. 6.4. Let the Gaussian centred at frequency ω_δ be denoted by $\mathcal{F}(\omega, \omega_\delta)$. The power spectrum of the spatially separated samples will then be a convolution of the Gaussian with the delta function frequency responses of the sample.

$$P(\omega) \rightarrow \int_0^\infty d\omega \mathcal{F}(\omega, \omega_\delta)\Big\{\delta(\omega_\delta - \omega_1) + \delta(\omega_\delta - \omega_2)\Big\}$$

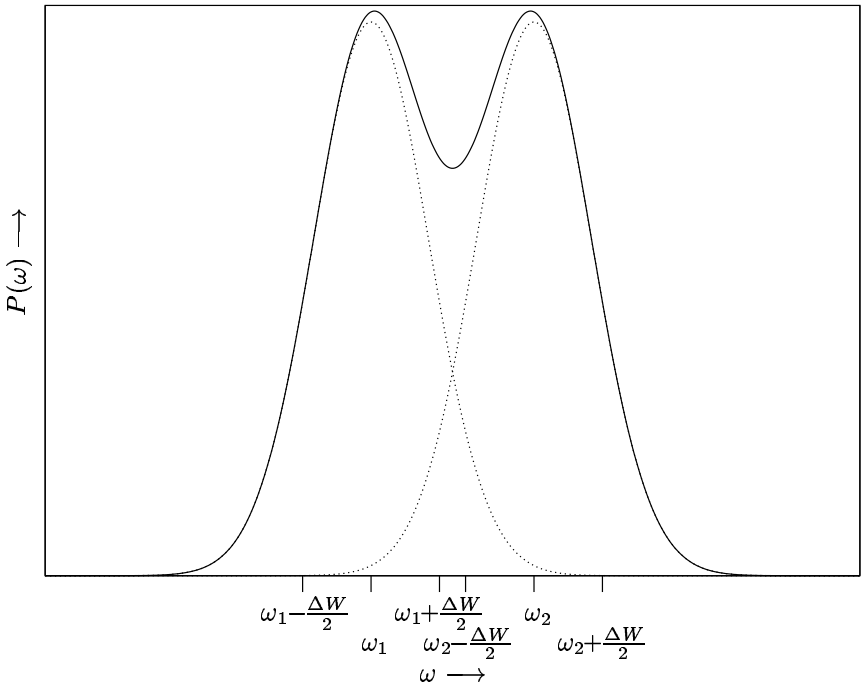

Figure 6.5: The power spectrum generated by a sample consisting of two spatially separated point-like spin populations. The spectrum is a convolution of the delta function frequency response of the sample and the Gaussian broadening by field inhomogeneities and spin-spin interactions.

Figure 6.5 indicates one of the key experimental limitations. In order to claim sharp resolution of the sample in question, the difference in resonance frequencies $\Delta\omega = \omega_2 - \omega_1$ must significantly exceed the Gaussian broadening of the signals; $\Delta\omega \gg \Delta W$. This places a constraint upon the *minimum allowable gradient*. By defining the lowest acceptable spatial resolution as $\Delta z_{max} = z_2 - z_1$, Eqn. 6.4 yields

$$\Delta z_{max} \sim \frac{\Delta W}{\gamma G} \tag{6.5}$$

Equation 6.5 suggests that one can obtain arbitrarily high spatial resolution $\Delta z \rightarrow 0$ by turning up G to an arbitrarily large value. However, there is a limit imposed by the measurement electronics, and one must choose G to satisfy the FOV criterion. In this experiment, a digitizing oscilloscope will be used to measure the power spectrum. This device will have a sampling frequency F_s and corresponding Nyquist limit

$$F_{Nyquist} = \frac{F_s}{2} \tag{6.6}$$

The values of G and ω_{rf} must be chosen such that the measured resonance frequency at any point in the sample after mixing $\omega_{measured} = |\omega_{resonance} - \omega_{rf}|$ does not exceed $2\pi F_{Nyquist}$ (Assume that B_o is not a variable - leave it fixed at the value of ~0.36 T used for the NMR experiment). One can use the gradient term in Eqn. 6.4 to establish the size of the FOV from the Nyquist frequency:

$$\text{FOV} = \frac{2\pi F_{Nyquist}}{\gamma G} \tag{6.7}$$

Therefore, setting the size of the FOV equal to the width of the region to be probed determines G.

Finally, one must then ensure that the resolution criterion given by Eqn. 6.5 is also satisfied by the value of G derived above. In this case the limiting quantity will be the digital sampling interval $T_s = 1/F_s$. To unambiguously resolve the difference in frequency between two sine waves, one of the waves must have a period *at least* $2T_s$ shorter than the other.

$$\frac{\omega_{measured}(z + \Delta z) - \omega_{measured}(z)}{2\pi} = \frac{1}{2T_s}$$

$$\Delta z = \frac{\pi}{\gamma G T_s} \tag{6.8}$$

To summarize, if one knows the sampling rate of the measurement system and the size of the sample to be probed, then the gradient can be chosen via Eqn. 6.7 and the theoretical resolution can be obtained from Eqn. 6.8. If the resolution surpasses the maximum acceptable value from Eqn. 6.5, then the performance of the imaging system can be deemed satisfactory.

6.3 Apparatus

The MRI experiment uses the NMR apparatus discussed in the previous chapter, but an additional set of coils for generating a gradient field along the $\hat{z}$ direction must be included. The additional coils are operated with the electrical current traveling in opposing directions in the two windings, and this configuration is referred to as a *Maxwell pair. The student is asked to calculate the theoretical gradient given that the elements of the Maxwell pair have 100 windings each, are* $d = 5.8\,cm$ *apart, and have radii of* $9.25\,cm$*. Determine* G *for currents ranging from 0 to* $10\,A$.

A *Tektronix model 360* digitizing oscilloscope will be supplied for this experiment. This device has a built-in power spectrum function and a floppy drive for exporting either data or images. Keep in mind that the power spectrum function displays the amplitude data on a logarithmic scale (dB), and so the width of peaks must be gauged from the -3dB points. Manuals will be provided with the oscilloscope and they will be necessary to quantify the performance of the device.

6.4 Samples and Data Analysis

A special ceramic sample vial that has two bores separated by roughly $3.3\,mm$ will be found with the apparatus. A very small drop of glycerin has been injected into each bore. Treat this sample as the idealized spatially separated spin populations discussed previously.

Begin the experiment with the gradient field turned off. Locate the resonance frequency ω_o and adjust the oscilloscope scale to fully resolve a FID from a roughly $\pi/2$ pulse. Detune the oscillator from resonance by roughly $30\,kHz$ to obtain a usable mixer signal. Note that the amount of detuning may need adjustment once the gradient is applied - one cannot discern the sign of $\omega_{measured} = |\omega_{resonance} - \omega_{rf}|$. An optimized system will have a resonance corresponding to $\omega_{measured} = 0$ on one extreme of the sample and $\omega_{resonance} < 2\pi F_{Nyquist}$ on the other extreme.

The $\pi/2$ pulse used to excite the spin system previously should be able to excite the spin populations in both samples with the gradient field turned on. Even though the continuous wave rf field has one specific frequency, the finite duration pulse will have a broad power spectrum. *It is recommended that the student measure the duration of the* $\pi/2$ *pulse supplied to the rf coils and to measure its power spectrum.* Note that the Fourier transform of a finite duration pulse is an elementary problem that the student has most likely encountered in either a mathematical physics or optics course.

Since the spatial separation of the spin populations is known, one can use this information and the observed difference in resonance frequencies, $\Delta\omega$ in Fig. 6.5, to calculate the gradient G. Compare with the results from a theoretical expression for the gradient. Once the gradient calibration has been completed, it should be possible to attempt to extract shape information from the measured intensity profile.

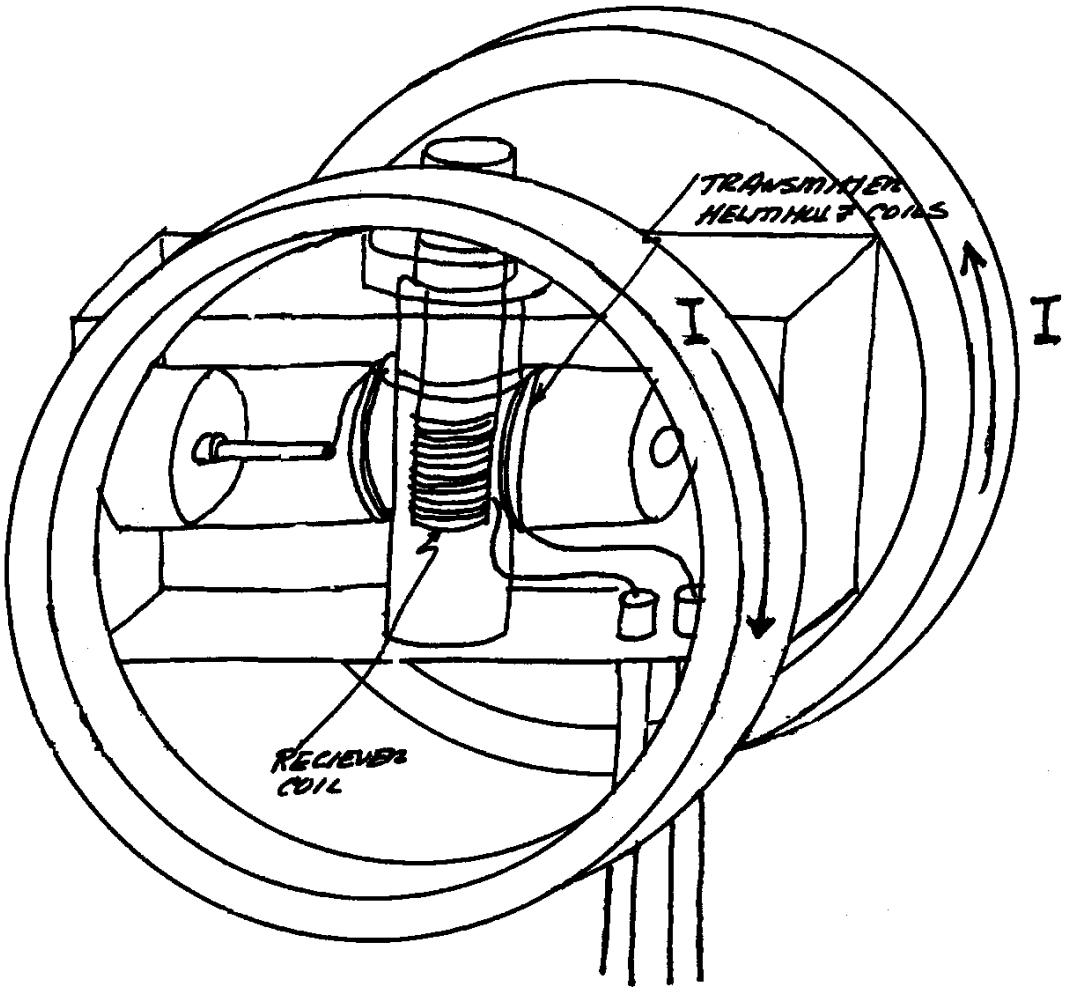

Figure 6.6: The central core of the MRI apparatus, showing how the gradient coils are oriented with respect to the rf transmitters and receiver.

6.5 Selected References

M.J. Bronskill and P. Sprawls, The Physics of MRI 1992 AAPM Summer School Proceedings. American Institute of Physics, New York, USA (1992)

This is a very nice presentation of MRI basics. It is written at an introductory level for graduate students, but should be very accessible to fourth year physics students as well.

P.C. Lauterbur, Nature **242**, 190 (1973)

The original suggestion that NMR be used for medical diagnostic imaging.

Thankyou to Dr. Alex MacKay and Joe Sihota (Physics 409, 1999-2000) for having done so much to make this experiment a reality.