Experiment 4

Thermal Noise

4.1 Introduction

Due to the random thermal motion of the charge carriers in a material, a fluctuating potential will be present across the terminals of any device which has electrical resistance. This fluctuating potential, which is small but quite measurable, is called thermal or Johnson noise. The mean value of the fluctuating potential is zero if averaged over a long period of time, however the voltage squared (hence the power dissipated) will be finite and can be measured with a suitable detector. Thermal noise places an ultimate limit upon signal to noise performance in real circuits.

4.2 The Nyquist Theorem

Using elementary arguments concerning the distribution of radiation in a one dimensional system, Nyquist argued that for a pure resistance R the mean square of the noise voltage $< V_n^2 >$ in a frequency increment Δf about the frequency f is given by

$$< V_n^2 >= 4k_B T R \, p(f) \Delta f \tag{4.1}$$

$$p(f) = \frac{hf/k_B T}{e^{hf/k_B T} - 1} \tag{4.2}$$

where $p(f)$ is a modified form of the Plank distribution, used to determine the thermal average energy in the radiation mode of frequency f. The student is asked to verify Eqn. 4.1 by consulting a reliable source.

At the frequencies and temperatures encountered in this experiment, $p(f)$ is very close to unity (given $T \sim 300$ K, at what frequency will this approximation fail?) and one is left with the most common form of Nyquist's theorem,

$$< V_n^2 >= 4k_B T R \Delta f \tag{4.3}$$

where the noise power per unit bandwidth is independent of frequency. Noise with this type of constant power spectrum is referred to as *white noise.*

From a practical standpoint, $< V_n^2 >$ is difficult to measure directly. Since the noise voltage is very small ($V_n \sim 10^{-7}$ V for $R \sim 1\ k\Omega$, $T \sim 300$ K and $\Delta f \sim 10\ kHz$), it must be amplified before it can be measured. For an amplifier whose frequency dependent voltage squared gain is $G(f)$, the noise voltage at the output of the amplifier will be

$$< V_o^2 >= 4k_B T R \int_0^\infty G(f)\, df \tag{4.4}$$

Note that despite the frequency dependence, Eqn. 4.4 has the same form as Eqn. 4.3: $< V_o^2 >= k_B T R \times$ *constant*. If one knows the *constant*, then a plot of $< V_o^2 >$ vs. R can be used to determine k_B (a constant that has been well established), thereby verifying Nyquist's theorem.

4.3 Experiment

The temperature of the resistor will differ from the ambient temperature because of the thermal timescale over which power is dissipated by the resistor. To avoid problems in trying to calibrate the change of resistor temperature against average power, a differential method is used; special purpose electronic circuits are available (RMS/DC converters) which provide a direct measure of $V_o(t)^2$, with the averaging time depending upon the device. The principles of operation of an RMS/DC converter can be found on the laboratory bench. The output from the converter is given by $V_{DC} = \langle V_o(t)^2 \rangle^{1/2}$ provided two key conditions are met:

- The output of the amplification system is linearly proportional to $\langle V_o(t)^2 \rangle^{1/2}$.
- The reactive component of the resistor's impedance is negligible over the bandwidth of the amplifier.

These two conditions will be verified during the course of the experiment.

The electronic system used in this experiment consists of a preamplifier (gain ~ 200) connected to a main unit containing a filer amplifier (used to set the pass band and gain) followed by an RMS/DC converter(whose conversion gain can also be set from the front panel). A block diagram of the apparatus is shown in Fig. 4.1 and a more involved schematic is shown in Fig. 4.2.

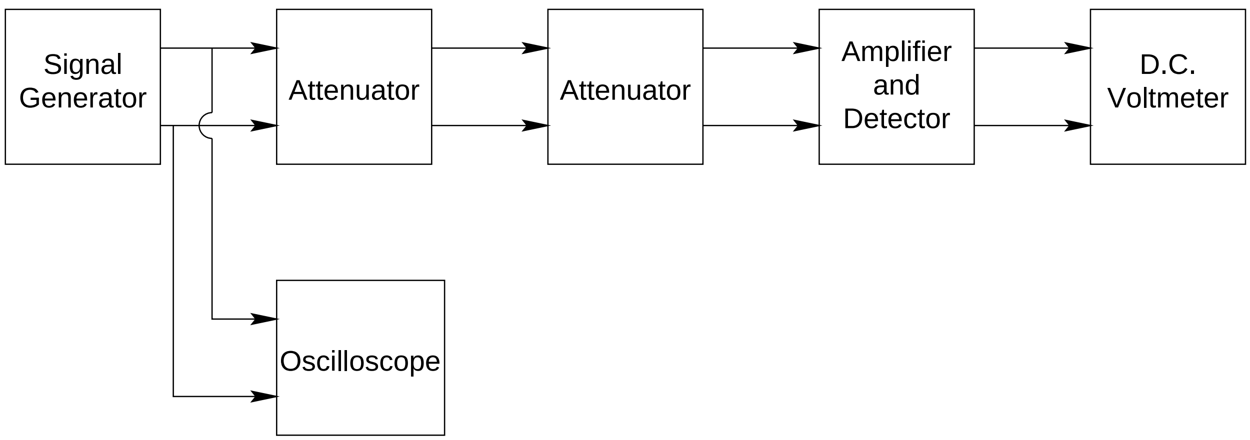

Figure 4.1: A block diagram of the thermal noise apparatus.

The amplifier response falls to zero at both high and low frequencies. Why? Both cutoffs are deliberate. The accuracy and interpretation of an experiment such as this depends entirely upon your quantitative knowledge of the amplification system. How linear is the amplifier? Does it clip large signals? Is it a strictly a voltage amplifier, and if not what limitations are thereby imposed? In this respect, it is essential to have a clear picture of the equivalent circuit of the amplifier.

- I. Measurement of Noise Voltage across a Carbon Composite Resistor

Connect various resistances (ranging up to 10 $k\Omega$) across the amplifier input, and measure the noise level with a DC voltmeter connected across the output of the amplifier. You should vary the amplifier bandwidth and note the changes in the plot of V_{DC}^2 vs R. Note which settings provide the most linear result. Throughout this work, it is essential to monitor the output of the linear amplifier on the oscilloscope - watch for interfering signals, clipping, etc...

The noise signal should be proportional to R, however you should notice that $V_{DC} \neq 0$ when $R = 0$. Why?

- II. Measurement of $\int_0^\infty G(f)df$

The next objective is to quantify the *constant* factor in Eqn. 4.4 by mapping out the frequency response of the circuit. This will be accomplished by supplying a signal of known amplitude and frequency to the amplifier circuit and then determining the gain at that frequency. The amplitude of the calibrated signal must be of the same order of magnitude as the measured thermal noise signals ($V_{DC} \sim 10^{-7}$ V); however, the *HP 3325A* function generator produces a minimum signal of 10^{-3} V. Hence the role of the *HP 355D* calibrated attenuator. Be certain that the attenuator and the preamplifier are properly terminated and that you understand why this is important. The output impedance of all *Hewlett Packard* equipment is 50 Ω, which is the recognized standard.

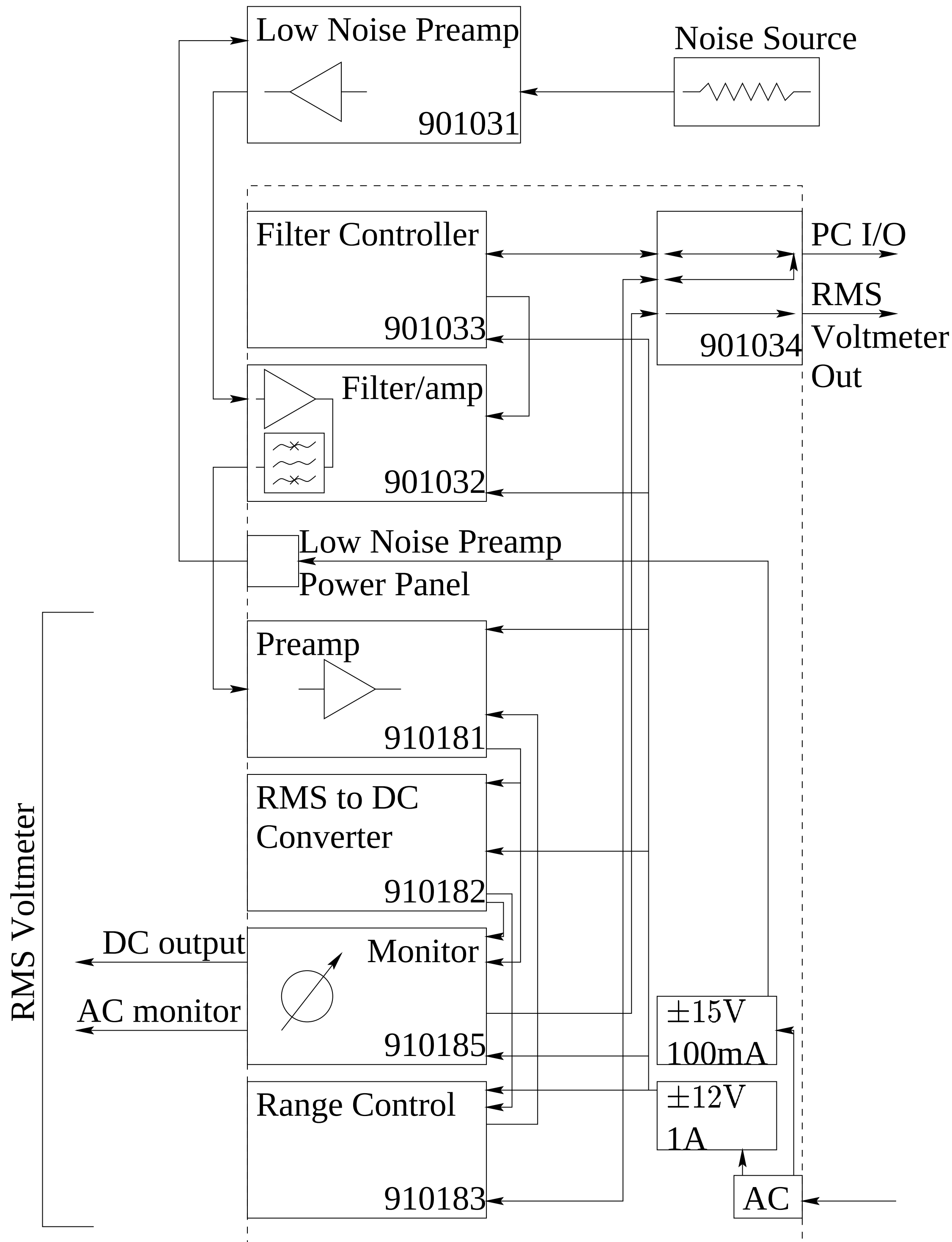

Figure 4.2: Schematic diagram of the thermal noise electronics.

Using the gain and bandpass settings which yielded the most linear plot of V_{DC}^2 vs R, determine $G(f)$ over a sufficiently broad frequency range to resolve the entire bandpass (a *Bode* plot will prove quite useful). Consider the nonzero intercept of V_{DC}^2 vs R when interpreting the plot of $G(f)$. It is also a worthwhile exercise to choose a few representative frequencies at which to measure the gain as a function of input voltage - this will verify the linearity of the amplifier and provide a more rigorous measure of $G(f)$ at those frequencies. Once you are satisfied with your measurement of $G(f)$, then calculate the *constant* $\int G(f)df$ to be used in Eqn. 4.4 using a suitable numerical method. This can in turn be used to calculate Boltzmann's constant, k_B.

4.4 Modeling the Amplifier Input

Recall that changing the amplifier bandwidth altered the curvature of V_{DC}^2 vs R. One possible source of error is the assumption that the input to the preamplifier has no role in the measurements. However, if one considers the construction of a typical FET, there are potential problems at high frequencies. In particular, the collection of a high density of charge carriers at a *pn* junction will give an effective capacitance, thus introducing an undesirable frequency dependence into the preamplifier's performance. A suggested model for this is shown in Fig. 4.3.

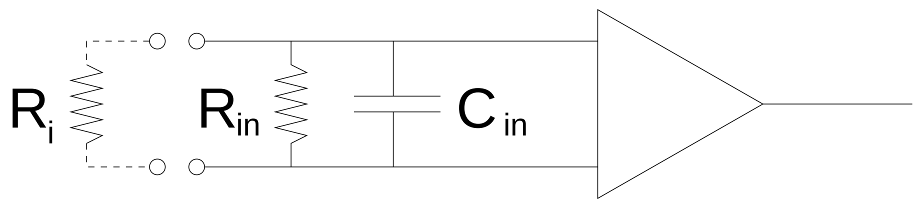

Figure 4.3: A plausible model for the amplifier input.

It is possible to measure R_{in} directly, however an accurate measure of C_{in} may prove to be difficult to obtain. An alternate course of action is to consider C_{in} as a free parameter, update the integration of $G(f)$ to account for the additional input impedance, and then fit to a plot of V_{DC}^2 vs R.

4.5 Selected References

H. Nyquist, Phys. Rev. 32:110, 1928

C. Kittel and H. Kroemer. *Thermal Physics*, W.H. Freeman and Company, San Fransisco USA, 1980.

Note that there is a mistake on page 101 (of the second edition). The line above Eqn. 30 should read "... of frequency $2\pi f_n = n\pi c'/L$ *from ..." and Eqn. 30 should be* $\delta f = c'/2L$*. Recall that modes of a transmission line are spaced* $\Delta\lambda = 2L$ *apart. Eqn. 32 should then be* $2\tau\,\Delta f/\delta f = 4\tau L\,\Delta f/c'$*.*