THE UNIVERSITY OF BRITISH COLUMBIA

Physics 122

Old $\mathcal{E}\&\mathcal{M}$ Examination Questions

1992-2001

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1. "QUICKIES"

(*a*) [1992 Final] The quantity $(\mu_0\epsilon_0)^{-1/2}$, where μ_0 and ϵ_0 are the two constants of electromagnetism, is equal to:

(*b*) [1992 Final] The oscillation frequency [in cycles per second] of a closed circuit consisting of an inductance L in series with a capacitance C is:

(*c*) [1992 Final] We have a velocity selector in which a magnetic field $\vec{B}$ is perpendicular to an electric field of 20,000 V/m. We find that charged particles with velocity $v = 4.0 \times 10^4$ m/s move through the device undeflected. What is the strength of $\vec{B}$ [in Tesla]?

(*d*) [1992 Final] What are the units for the flux of electric field through a surface, in terms of meters and volts?

(*e*) [1992 Final] If we have a closed surface inside which are 3 electric charges ($q_1 = +0.31$ C, $q_2 = -0.17$ C and $q_3 = +0.42$ C, what is the total electric flux through the surface? [Given: $(4\pi\epsilon_0)^{-1} = 8.99 \times 10^9$ N m^2/C^2.]

(*f*) [1992 Final] You have an alternating voltage supply, $V(t) = V_0 \sin \omega t$. What is the *rms* (root-mean-square) voltage in terms of V_0?

(*g*) [1993 Final] Maxwell completed the laws of electromagnetism by adding a crucial extra term to which of the following: [underline one] (*i*) Gauss' Law for electricity; (*ii*) Gauss' Law for magnetism; (*iii*) Faraday's Law; (*iv*) Ampere's Law.

(*h*) [1993 Final] Describe in one short sentence the purpose of the *magnet* in the operation of a cyclotron to accelerate protons:

(*i*) [1993 Final] Which of the following has the dimensions of *time?* [encircle one]

$$(i)\ LR \qquad (ii)\ CR \qquad (iii)\ LC \qquad (iv)\ \frac{1}{LR} \qquad (v)\ \frac{1}{RC} \qquad (vi)\ \frac{1}{LC}$$

(*j*) [1994 Final] Charges of $+Q$ and $-2Q$ ($Q > 0$) are located as shown:

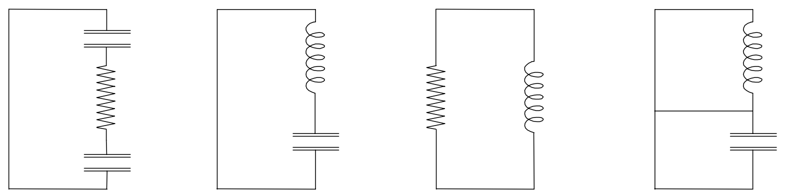
$$+Q \bullet \qquad\qquad\qquad \bullet -2Q$$

A point where the electric field could equal zero is located

(*i*) to the left of the $+Q$ charge (*ii*) between the two charges

(*iii*) to the right of the $-2Q$ charge (*iv*) nowhere

(*k*) [1994 Final] Encircle any of the following circuits that will *oscillate* if a current flows:



(*l*) [1994 Final] A proton (initially at rest) "bounces" off the magnetic fields of two galaxies travelling toward it from opposite directions, each with the same speed v. Assume that each "collision" between the proton and a galaxy is perfectly elastic and that all the velocities are parallel. If, after a total of 10 such collisions, the speed of the proton is 2×10^5 m/s, what is the value of v?

(*m*) [1994 Final] The idea that "lines of force" are continuous (unbroken) except where they begin and end on *charges* corresponds to which of the following Laws of electromagnetism? [underline one]

(*i*) Gauss' Law (*ii*) Faraday's Law (*iii*) Ampere's Law

(*n*) [1994 Final] Two coils have a net inductance of 4 mH when connected in series and 1 mH when connected in parallel. Their individual inductances are therefore $L_1 =$ ________ mH and $L_2 =$ ________ mH.

(*o*) [1994 Final] A cube 1 m on a side completely surrounds an electric dipole consisting of a positive electric charge of 0.01 C located 1 cm away from an equal-magnitude negative charge. What is the average value, taken over the whole surface, of the electric field normal to the surface?

(*p*) [1994 Final] An oscillator consisting of an inductance L, a capacitance C and a resistance of 1000 Ω, all connected in series, has a resonant frequency of $f_\circ = (1/2\pi)$ kHz and a decay time of $\tau = 0.1$ s. What is the value of the inductance?

(*q*) [1994 Final] If $\vec{\boldsymbol{g}}$ is the acceleration of a small test mass due to the gravitational attraction of a large accumulation of other mass, we can write GAUSS' LAW FOR GRAVITY in the form $\oint\!\!\oint \vec{\boldsymbol{g}} \cdot d\vec{\boldsymbol{A}} = k_G M_{\rm encl}$, where $M_{\rm encl}$ is the total mass inside the closed surface over which $\vec{\boldsymbol{g}} \cdot d\vec{\boldsymbol{A}}$ is integrated. Show that the constant $k_G = 4\pi G$, where G is the universal gravitational constant.

(*r*) [1995 Final] Show with sketches how to combine

i. two identical capacitors to make an equivalent capacitance half as big;

ii. two identical inductances to make an equivalent inductance half as big.

(*s*) [1995 Final] What is the *direction* of any electric field $\vec{\boldsymbol{E}}$ just outside the surface of a conductor, and *why?*

(*t*) [1995 Final] Explain briefly why static magnetic fields cannot change the *energy* of a charged particle, no matter how much they may alter its direction of motion.

(*u*) [1995 Final] The idea that the "circulation" of magnetic field "lines" around an *open* surface is proportional to the net electric current flowing through that surface corresponds to which of the following Laws of electromagnetism? [encircle one]

(*i*) Gauss' Law (*ii*) Faraday's Law (*iii*) Ampere's Law

(*v*) [1996 Final] If I have an inductance of 3×10^{-6} H and a capacitance of 3×10^{-6} F, the frequency of oscillation of the two elements combined in a circuit is ________________ Hz.

(*w*) [1996 Final] I have crossed magnetic and electric fields and I observe that protons of velocity 4.0×10^6 m/s, perpendicular to both fields, pass through the system undeflected. If the electric field strength is 35,000 V/m, the magnetic field strength must be ________________ T.

(*x*) [1997 Final] Which of the following has the dimensions of time? [**encircle one**]

(i) LR (ii) CR (iii) LC (iv) $1/LR$ (v) $1/RC$ (vi) $1/LC$

(*y*) [1997 Final] For an AM radio we have a fixed inductance, L, and a variable capacitance, C, to tune to the stations of the AM band. If $C = 3.0 \times 10^6$F for the upper end of the band (1600 kHz), what value of C tunes to the lower end of the band (530 kHz)?

(*z*) [1997 Final] Maxwell completed the laws of electromagnetism by adding a crucial extra term to which of the following: [**underline one**]

(i) Gauss' Law for electricity; (ii) Gauss' Law for magnetism; (iii) Faraday's Law; (iv) Ampère's Law.

2. "NOT QUITE SO QUICKIES"

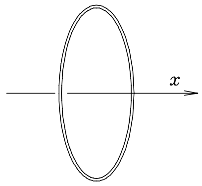
(*a*) [1993 Final] A very oddly shaped surface completely surrounds three electric charges, $q_1 = 13 \times 10^{-9}$ C, $q_2 = -7 \times 10^{-9}$ C and $q_3 = 8 \times 10^{-9}$ C. If the total area of the surface is 6 m^2, what is the average value, taken over the whole surface, of the electric field normal to the surface? ________________________ V/m. ($\epsilon_0 = 8.85 \times 10^{-12}$ C^2/N-m^2.)

(*b*) [1993 Final] An electric field of 136 V/m acts between two parallel capacitor plates with air between them. Without altering the charge on either plate, we now completely fill the space between the plates with strontium titanate, whose dielectric constant is $\kappa = 310$. What is then the electric field between the plates? ________________________ V/m.

(*c*) [1993 Final] Two resistors have a net resistance of 16 Ω when connected in series and 3 Ω when connected in parallel. Their individual resistances are therefore $R_1 =$ ________________ Ω and $R_2 =$ ________________ Ω.

(*d*) [1993 Final] An oscillator produces a signal of 15 V amplitude and variable frequency; it is connected to a circuit with a 30 Ω resistance and an unknown capacitance and inductance. If the frequency is "tuned to resonance" the maximum current flowing through the resistance will be ____________ A.

3. **Ring of Charge [1992 Midterm]**
For a uniform ring of electric charge, the electric field on the x-axis through the centre of the ring is given by

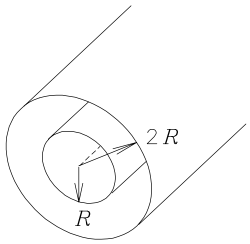


$$E = kQx/(a^2 + x^2)^{3/2}$$

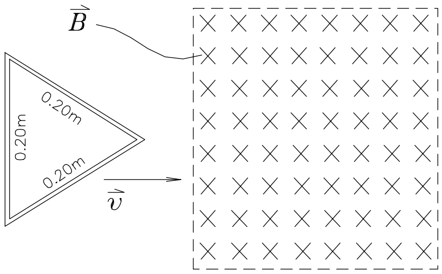
(*a*) Describe in a few words what the quantities k, Q and a are.

(*b*) If we place a small charge q (opposite in sign to Q), with mass m, very near the ring centre an on the x-axis, what is the period of simple harmonic oscillations of the charge q for oscillations of amplitude very small compared to the ring radius? Give your answer in terms of m, q, k, Q and a.

4. **Charged Cylinders [1992 Final]**
Two concentric cylindrical shells of insulating material are arranged as shown; the inner cylinder has a radius $R = 0.3$ m and the outer cylinder's radius is $2R$. Each cylinder is uniformly charged, the inner one with -0.4×10^{-10} C/m^2 and the outer one with $+0.9 \times 10^{-10}$ C/m^2. Plot the electric field [in V/m] as a function of the distance r from the central axis, from $r = 0$ to $r = 4R$. [Given: $(4\pi\epsilon_0)^{-1} = 8.99 \times 10^9$ N m^2/C^2.]



5. **Triangular Loop [1992 Final]**
A wire loop in the shape of an equilateral triangle (each side 0.20 m long) travelling at a constant speed $v = 5.0$ m/s moves, pointy-end first, into a region where a uniform magnetic field $B = 0.40$ T points into the paper, as shown.



(*a*) Indicate on the diagram the direction of current flow in the triangular loop as it enters the field.

(*b*) What is the maximum induced $\mathcal{EMF}$ around the loop as it enters the field.

(*c*) Sketch the induced $\mathcal{EMF}$ around the loop as a function of time, from the time it begins to enter the field until it is entirely in the field.

6. **Earth's-Field Cyclotron [1992 Final]** We want to build a cyclotron in Vancouver to produce 8 MeV protons. The cyclotron will have no special magnet of its own; instead, it will use the Earth's magnetic field ($B = 5 \times 10^{-5}$ T). [Given: the mass of a proton is $m_p = 1.67 \times 10^{-27}$ kg; the charge of a proton is $q_p = 1.6 \times 10^{-19}$ C; and 1 MeV$= 1.6 \times 10^{-13}$ J.]

(*a*) What is the orbit radius of the protons at 8 MeV?

(*b*) What is the frequency (in Hz) of the RF electric field we must supply to the cyclotron "dees?"

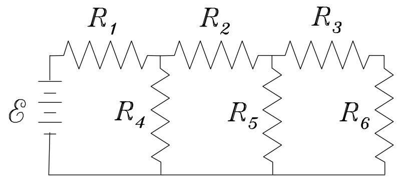
7. **RLC Circuit [1992 Final]** First you fully charge a capacitor ($C = 6.4\ \mu$F) with a 12 V battery. Then you disconnect the battery and discharge the capacitor through a resistor ($R = 3$ Ohms) and an inductor ($L = 2.0 \times 10^{-3}$ Henry) in series.

(*a*) What is the charge on the capacitor when you begin discharging at $t = 0$?

(b) How long does it take for the charge *amplitude* to fall to one tenth of its initial value?

(c) At what time does the charge *first* fall to one tenth of its initial value?

8. **Resistors [1992 Final]**
We are given a battery ($\mathcal{E} = 12$ V) and six identical resistors: $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 13\ \Omega$ which are connected as shown in the diagram.



(a) How much current flows through R_6?

(b) What is the total resistance of the circuit?

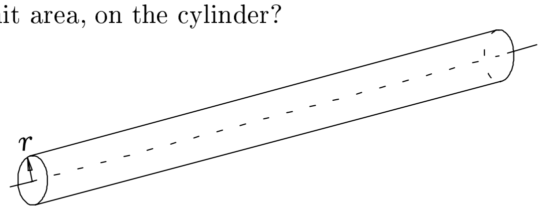
9. **Solenoid [1992 Final]** You wind 1 km of copper wire (resistivity $\rho = 1.7 \times 10^{-8}\ \Omega$m, wire diameter $d = 0.9$ mm) into a solenoid of length $\ell = 1.1$ m and diameter $D = 0.1$ m. [Given: $\mu_0 = 10^{-7}$ T m/A.]

(a) How much current flows through the solenoid a long time after you connect it to a 12 volt battery?

(b) What is then the magnetic field inside the solenoid?

(c) What is the total energy stored in the solenoid under these conditions?

10. **Coaxial Cable [1993 Midterm]** A very long straight wire is uniformly negatively charged with -3.6×10^{-9} Coulomb/m. The wire is surrounded by a thin cylindrical shell of radius $r = 1.5$ cm, coaxial with the wire, which is uniformly positively charged such that the electric field outside the cylinder is zero. What is σ, the charge per unit area, on the cylinder?



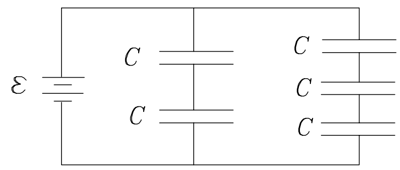
11. **Electromagnetic Force [1993 Final]** In a certain region of space we have an electric field $\vec{\boldsymbol{E}}$ and a magnetic field $\vec{\boldsymbol{B}}$:

$$\vec{\boldsymbol{E}} = [\,7\hat{\imath} + 4\hat{\jmath} + 6\hat{k}\,]\ \mathrm{V/m} \qquad \vec{\boldsymbol{B}} = [\,-3\hat{\imath} - 4\hat{\jmath} + 10\hat{k}\,]\ \mathrm{T}$$

If an electron is moving in this region with a velocity $\vec{\boldsymbol{v}} = [\,2\hat{\imath} + 3\hat{\jmath}\,]$ m/s,

what is the instantaneous force (full vector form by components) on the electron? ($e = 1.6 \times 10^{-19}$ C.)

12. **Capacitors [1993 Final]**
Five identical capacitors, each with $C = 9\ \mu F$, are connected as shown to a battery with $\varepsilon = 12\ V$. If the capacitors are initially uncharged, how much charge flows out of the battery to fully charge the capacitors?



13. **RL Circuit [1993 Final]** We have used a battery to build up a magnetic field of $B = 0.6$ T inside a superconducting coil 0.5 m long and 0.02 m^2 in cross-sectional area. Now we suddenly switch off the battery and put the coil in series with a resistance of 120 Ω. We observe that it takes 3.6 milliseconds (ms) for the field in the coil to drop to 0.1 T. ($\mu_0 = 4\pi \times 10^{-7}$ T-m/A.)

(a) What is the total number of turns in the coil?

(b) How many *joules* of energy are stored in the coil at $t = 0$?

(c) How long does it take for the stored energy to fall to half its initial value?

(d) What is the power dissipated in the resistance 1 ms after the battery is switched off?

14. **Generator [1993 Final]** Some energetic students are "flipping" a large circular coil (radius $r = 5$ m, $N = 1000$ turns) about an axis perpendicular to the Earth's magnetic field in Vancouver. The maximum B field through the coil is 0.3×10^{-4} T. If the students crank furiously but smoothly at a steady rate of 2 rotations per second, *find:*

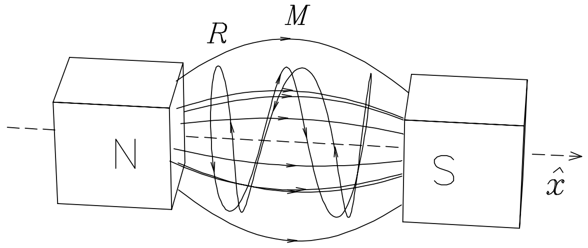
   (*a*) the maximum current that flows from this generator through the 100 Ω resistance of its own wires, if we connect the ends of the coil together;

   (*b*) the average power dissipated in said 100 Ω resistance;

   (*c*) the diameter of the copper wire (circular cross section) from which the coil is wound. (The resistivity of copper is 1.7×10^{-7} Ω-m.)

15. **Magnetic Mirror [1993 Final]**

   A proton is trapped in a magnetic mirror system as shown. In the spiralling motion the total kinetic energy of the proton and the magnetic flux encircled by its orbit both remain constant throughout its motion.



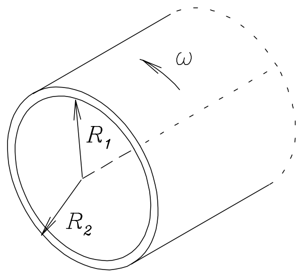
   (*a*) Explain why the total kinetic energy of the proton remains constant.

   (*b*) At the midpoint M between the two magnets the component of the proton's velocity *parallel* to the $\hat{x}$ direction (along the line joining the centres of the magnet pole tips, as shown) is $\frac{1}{4}$ of the velocity perpendicular to $\hat{x}$. What is the ratio of the magnetic field strength at the midpoint M to that at the reflection point R?

16. **Electrostatic Gaiacide [1994 Midterm]** Suppose that some evil beings start to "seed" the Earth (also known as *Gaia*) with extra *protons* distributed evenly to give the planet a uniform positive charge per unit volume. [Treat the Earth as an insulator with respect to protons — *i.e.* the protons are *not* able to move freely.] *How many* protons can the Earth hold before it blows up and flies apart into space?

17. **A Really BIG Space Colony [1994 Final]**

   Far in the future, a space colony is built in the form of a long, uniform cylindrical shell of inner radius $R_1 = 0.9R_E$ and outer radius $R_2 = R_E$ (where $R_E = 6367$ km is the Earth's radius). In the following questions, assume that you are far from the ends so that it looks essentially like an infinitely long cylinder.



   (*a*) What must be the period T of rotation of the cylinder about its axis [in *minutes*] in order to produce an *apparent* gravitational acceleration at the *inner* surface that is equal in magnitude to the actual acceleration of gravity at the surface of the Earth, $g_\circ = 9.81$ m/s^2.

   (*b*) Show that the cylinder must be composed of a material that is *3.5088* times as dense as the mean mass density of the Earth $\left(\rho_E = \dfrac{M_E}{\frac{4}{3}\pi R_E^3}\right)$ if the actual acceleration of gravity at the *outer* surface is to be equal to $g_\circ$.

   (*c*) If the cylinder rotates with the period calculated in the first part and the mass density has the value specified in the second part, calculate the magnitude and direction of the *apparent* acceleration of gravity at the *outer* surface.

18. **Capacitor [1994 P115 Final]** A charged parallel-plate capacitor is *isolated* so that the charge cannot leak off. The volume between the plates is 60.0×10^{-6} m^3 and the energy density in this volume is 0.20×10^{-3} J/m^3. [Given: $\epsilon_\circ = 8.85 \times 10^{-12}$ F/m.]

   (*a*) Calculate the electric field between the plates.

The separation between the plates is now increased slowly until the volume has tripled, while the capacitor remains isolated.

(*a*) What is the ratio of the new electric field to the original electric field? [Explain!]

(*b*) What is the ratio of the new voltage to the original voltage? [Explain!]

(*c*) Calculate the work done in separating the plates.

(*d*) Explain in detail how you would measure the dielectric constant of a flat slab of plastic.

2. **A Copper Coil [1994 Final]** A volume $V = 1.5 \times 10^{-2}$ litres of solid copper (resistivity $\rho = 1.7 \times 10^{-8}$ Ω-m) is made into a wire with a circular cross section 6×10^{-4} m in diameter.

(*a*) What is the resistance R of the wire?

(*b*) If this wire is used to make a circular solenoid of length $\ell = 0.4$ m and inner diameter 0.020 m, show that the inductance of the solenoid will be $L = 0.7036$ mH. [Given: $\mu_\circ = 4\pi \times 10^{-7}$ T-m/A.]

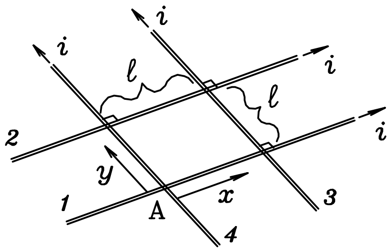
(*c*) If you now connect this solenoid to a 9 V battery and wait until the current reaches its maximum value (assume that the only resistance in the circuit is that of the wire), what is the energy stored in the solenoid?

(*d*) If you now disconnect the battery, simultaneously shorting out the solenoid so that it discharges through its own resistance, what is the "half-life" of the current (*i.e.* the time taken for the current to fall to one-half its initial value)?

(*e*) Derive an expresion for the above "half-life" which depends only on $\mu_\circ$, ρ, the volume of copper V and the length ℓ of the solenoid. (Perhaps surprisingly, for a given V and ℓ the half-life does *not* depend on the radius r of the solenoid, the diameter d of the wire or the number N of turns in the solenoid.)

3. **Crossed Wires [1994 Final]**

Four identical straight wires, each carrying a current of $i = 1$ A, cross at right angles in a common plane but do not make electrical contact with one another. Each pair of wires is a distance $\ell = 1$ cm apart. Consider all the wires to be infinite in length.
[Given: $\mu_\circ = 4\pi \times 10^{-7}$ T-m/A.]



(*a*) What is the magnetic field at wire *2* due to the current in wire *1*?

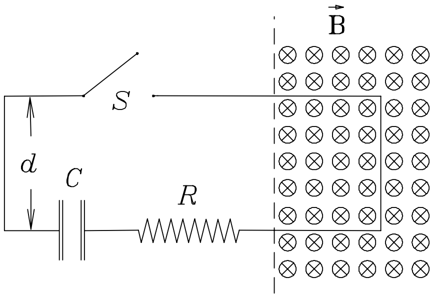
(*b*) What is the *force per unit length* exerted on wire *2* by wire *1*?

(*c*) If the point of intersection **A** is used as an origin and the coordinates x and y of a point in the plane of the wires are measured from **A** as shown, derive an expression for the magnetic field *anywhere in the plane of the wires* as a function of x and y (expressed in terms of ℓ, i and $\mu_\circ$). Be sure to specify the *direction* of $\vec{B}$.

(*d*) Can this arrangement actually be maintained with real wires? (Explain your reasoning! No credit will be given for a correct guess with the wrong explanation.) [HINT: What forces are exerted on wire *4* by wire *1*?]

4. **Circuit in a Field [1994 P115 Final]**

Consider a rectangular loop of wire as shown containing a switch S, a capacitance $C = 0.500$ μF and a resistance $R = 1.00 \times 10^4$ Ω. The uniform magnetic field $\vec{B}$ (into the page) has a constant magnitude $B = 0.200$ T to the right of the dashed line and zero to its left. The distance $d = 14.0$ cm.



(*a*) Suppose the loop is held fixed in place and the voltage across the capacitor is initially 50.0 V. If the switch is closed at time $t = 0$, determine the time t_A for the current to fall to 3.00×10^{-3} A.

(*b*) What is the force on the loop at $t = t_A$ if the current flows in a clockwise direction?

After the capacitor has completely discharged, a force is applied to pull the loop to the left, out of the magnetic field, at a constant velocity $v = 0.700$ m/s.

(a) Calculate the magnitude of the induced $\mathcal{EMF}$.

(b) This $\mathcal{EMF}$ will cause a current to flow which will charge the capacitor. Indicate which capacitor plate will become positively charged and give your reasoning.

3. **RC Circuit [1995 Midterm]** A capacitor ($C = 8$ mF) is initially charged to *half* of the full charge Q it would have if had been connected to the battery ($\mathcal{E} = 6$ V) for a very long time. The circuit contains a resistance ($R = 2\ \Omega$). At $t = 0$ the switch S is closed. *How long does it take* for the capacitor to reach $\frac{7}{8}$ of its full charge Q?

4. **Orbit Around a Charged String [1995 Midterm]** A long straight cylindrical "cosmic string" in interstellar space is charged uniformly with a negative electric charge $-\lambda$ per unit length. The resultant electric field acts on a particle of mass m and positive charge q which is in a circular orbit around the string at a radius r. (We assume the string itself has a still smaller radius. We also ignore any possible effects of the string's gravity.)

   (a) Show that the *kinetic energy* of the particle is given by $\frac{1}{2}mv^2 = k_E q\lambda$ (where $k_E \equiv 1/4\pi\epsilon_\circ$), regardless of the value of m or r.

   (b) If the charge on the wire is $\lambda = 1.0$ C/m and the particle has a charge of $q = 1.0$ C and a mass of 0.2 kg, show that the particle is moving at 0.1% of the speed of light.

   (c) If the radius of the particle's orbit is $r = 0.1$ m, what is the *magnetic field* at the centre of its orbit due to the circulating charge?

5. **Athletic Potential [1995 Midterm]** A sprinter on the UBC track team runs at a speed of 8 m/s while holding a 5 m long pole vault pole horizontal at right angles to her direction of motion.[1] The pole has a fine copper wire down the center from one end to the other. If the Earth's magnetic field at the location of the runner has a magnitude of 0.3×10^{-4} T and makes an angle of 45° with the horizontal, *what is the potential difference* [in volts] between one end of the pole and the other?

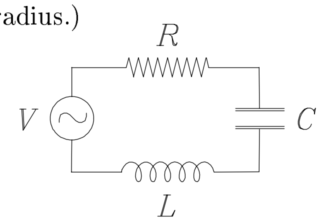
6. **Gauss' Law and Flat Earth [1995 Final]** If $\vec{g}$ is the acceleration of a small test mass due to the gravitational attraction of a large accumulation of other mass, we can write GAUSS' LAW FOR GRAVITY in a form analogous to that for electrostatics, with the replacements $\vec{E} \to \vec{g}$ and $Q_{\rm encl}$ (charge within closed surface) $\to M_{\rm encl}$ (mass within closed surface).

   (a) Write down the general form of GAUSS' LAW FOR GRAVITY, including the correct constants and defining any terms not explained above.

   (b) Using the above LAW, derive a formula for the acceleration of gravity outside an infinite flat slab of mass containing σ_m mass per unit surface area.

   (c) If a technologically advanced civilization were able to construct such a flat slab — not truly infinite in area, of course, but large enough that "edge effects" could be ignored near the middle of the slab — out of a material with a uniform mass density ρ_m per unit volume equal to the mean ρ_m of the Earth, *how thick must the slab be* in order to produce the same surface gravity as we feel at the surface of the Earth? (Express your answer in units of R_E, the Earth's radius.)

7. **AC Circuits [1995 Final]**
   The circuit shown is driven by an AC power supply generating $V(t) = V_\circ \sin\omega t$, where $V_\circ = 150$ Volts and $\omega = 2\pi \times 60$ Hz. This voltage is applied to a resistance R, a capacitor C and an inductance L, connected in series.



   (a) If $L = 0$, $C = 0$ and $R = 10.0\ \Omega$, what are the values of the *maximum* current i_m and the *average* current $\bar{\imath}$ in the circuit?

[1]This is not the usual orientation in which a pole vaulter would hold the pole during a "run up," but this is a sprinter performing a Physics 122 experiment, so unconventional behaviour is expected!

(*b*) If $R = 10.0\ \Omega$ and $C = 6.00\ \mu$F, what value of the inductance L will give the largest possible amplitude of current oscillations in the circuit?

(*c*) With the values of R, C and L given (or calculated) in the preceding part, what is the average *power* dissipated in the circuit?

8. Electromagnetic Waves [1995 Final] A plane electromagnetic wave (in which the $\vec{E}$ and $\vec{B}$ fields vary only in the direction of propagation) has its electric field in the x direction and its magnetic field in the z direction.

(*a*) Based only on the above information, what can you say about the *direction of propagation* of the wave?

(*b*) If at one position in space $E_x = E_\circ \sin \omega t$ and $B_z = B_\circ \sin \omega t$, with $E_\circ = 40.0$ V/m and $B_\circ = 1.334 \times 10^{-7}$ T, what is the *total energy U* contained in *one cubic kilometer* of such a plane wave? (Assume that the wavelength is very small compared to a kilometer.)

(*c*) The speed of light c is given in terms of the permittivity of free space $\epsilon_\circ$ and the permeability of free space $\mu_\circ$ as

$$c = \frac{1}{\sqrt{\mu_\circ \epsilon_\circ}}.$$

Prove that the right hand side of this equation has units of velocity.

9. Electromagnetic Force [1996 Final] At $t = 0$ an electron ($q = -1.602 \times 10^{-19}$ C, $m = 9.11 \times 10^{-31}$ kg) is at position $(x, y, z) = (0, 0, 0)$ and moving with velocity 3.0×10^6 m/s along the positive x-axis, in a region of space where the electric field (33 V/m) and the magnetic field (1.30×10^{-5} T) are both uniform and pointing along the positive z-axis.

(*a*) What is the speed of the electron at $t = 1.2 \times 10^{-6}$ s?

(*b*) What is the position of the electron (x, y and z-coordinates) at $t = 1.2 \times 10^{-6}$ s?

10. Resistance [1996 Final] We have seven equal resistances (R = 120 Ω) connected as shown to a battery ($\varepsilon = 12$ V). How much current flows out of the battery?

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11. LC Circuit Time-Dependence [1996 Final] In an LC circuit with $C = 90\ \mu$F, the current is given as a function of time by $i = 3.4\ cos(1800\ t + 1.25)$, where t is in seconds and i is in amperes.

(*a*) [4 marks] How soon after $t = 0$ will the current reach its maximum value?

(*b*) [4 marks] Calculate the inductance.

(*c*) [4 marks] Find the total energy in the circuit.

12. A.C. Circuit [1996 Final] We have an oscillator ($V = V_m sin\omega t$, with $V_m = 120\ V$ and frequency = 60 cycles/sec.) connected in series to a resistance ($R = 30\ \Omega$) an inductance ($L = 0.050$ H) and a capacitance ($C = 65\ \mu$F).

(*a*) [6 marks] What is the maximum current in the circuit?

(*b*) [4 marks] What is the average power delivered to the circuit?

13. ELECTROMAGNETIC FORCE [1997 Final] In a certain region of space we have an electric field $\vec{E}$ and a magnetic field $\vec{B}$:

$$\vec{E} = [7\hat{\imath} + 4\hat{\jmath} + 6\hat{k}]\ \text{V/m} \qquad \vec{B} = [-3\hat{\imath} - 4\hat{\jmath} + 10\hat{k}]\ \text{T}$$

If an electron is moving in this region with a velocity $\vec{v} = [2\hat{\imath} + 3\hat{\jmath}]$ m/s,
what is the instantaneous force (full vector form by components) on the electron? ($e = 1.6 \times 10^{-19}$ C).

14. CAPACITANCE [1997 Final] We attach a battery, $\varepsilon = 12$ volts, to 9 equal capacitances in a circuit as shown, with $C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = C_7 = C_8 = C_9 \equiv C = 9.0 \times 10^{-6}$ F.

(a) What is the amount of charge which flows from the battery to fully charge the system of capacitors?

(b) What is the charge on each of the capacitors?

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15. RL CIRCUIT [1997 Final] We have used a battery to build up a magnetic field of $B = 0.6$ T inside a superconducting coil 0.5 m long and 0.02 m^2 in cross-sectional area. Now we suddenly switch off the battery and put the coil in series with a resistance of 120 Ω. We observe that it takes 3.6 milliseconds (ms) for the field in the coil to drop to 0.1 T. ($\mu_0 = 4\pi \times 10^{-7}$ Tm/A.)

    (a) What is the total number of turns in the coil?

    (b) How many *joules* of energy are stored in the coil at $t = 0$?

    (c) How long does it take for the stored energy to fall to half its initial value?

    (d) What is the power dissipated in the resistance 1 ms after the battery is switched off?

16. GENERATOR [1997 Final] Some energetic students are "flipping" a large circular coil (radius $r = 5$ m, $N = 1000$ turns) about an axis perpendicular to the Earth's magnetic field in Vancouver. The maximum B field through the coil is 0.3×10^{-4} T. If the students crank furiously but smoothly at a steady rate of 2 rotations per second, find:

    (a) the maximum current that flows from this generator through the 100 Ω resistance of its own wires, if we connect the ends of the coil together;

    (b) the average power dissipated in said 100 Ω resistance;

    (c) the diameter of the copper wire (circular cross section) from which the coil is wound. (The resistivity of copper is $1.7 \times 10^{-8} \Omega$m.)

17. DAMPED OSCILLATIONS [1997 Final] We have an RLC series circuit, as shown, with $R = 10\ \Omega$, $L = 10^{-4}$ H, $C = 10^{-6}$ F.

    (a) Write down the circuit equation which gives the voltage drops around the entire circuit.

    (b) If we start with a charged capacitor how long does it take for the amplitude of the oscillating current to fall to half of its original value?

    (c) What is the frequency of the charge oscillations?

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— *FINIS* —