THE UNIVERSITY OF BRITISH COLUMBIA

Physics 108

SOLUTIONS

MIDTERM - 7 March 2003

TIME: 50 MINUTES

INSTRUCTOR(S): Jess H. Brewer

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1. QUICKIES [10 marks each — 60 total]

(*a*) Under what circumstances would the *entropy* of a system *decrease* with the *addition of energy*, and what could you say about the *temperature* of such a system?
ANSWER: In order for there to be less ways to redistribute more energy, there must be some limit to how much energy the system can hold. (For instance, the system of N spins in a magnetic field.) Then the options must get more limited when that limit is approached. Such a system has a negative slope of the entropy as a function of energy, which means a negative inverse temperature and in turn a negative temperature (which, as we all know, is hotter than even infinite temperature).

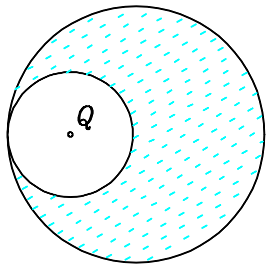
(*b*) Charges of $+Q$ and $-2Q$ are located as shown:

$$+Q \bullet \qquad\qquad\qquad \bullet -2Q$$

A point where the electric field could equal zero is located (encircle your choice)

$\boxed{(i) \text{ to the left of the } +Q \text{ charge}}$	(ii) between the two charges
(iii) to the right of the $-2Q$ charge	(iv) nowhere

(*c*) A solid *conducting* sphere of radius R has a spherical cavity of radius $R/2$ just touching one side, as shown. In the exact centre of the *cavity* is a point charge Q. Describe in detail the *electric field as a function of position*

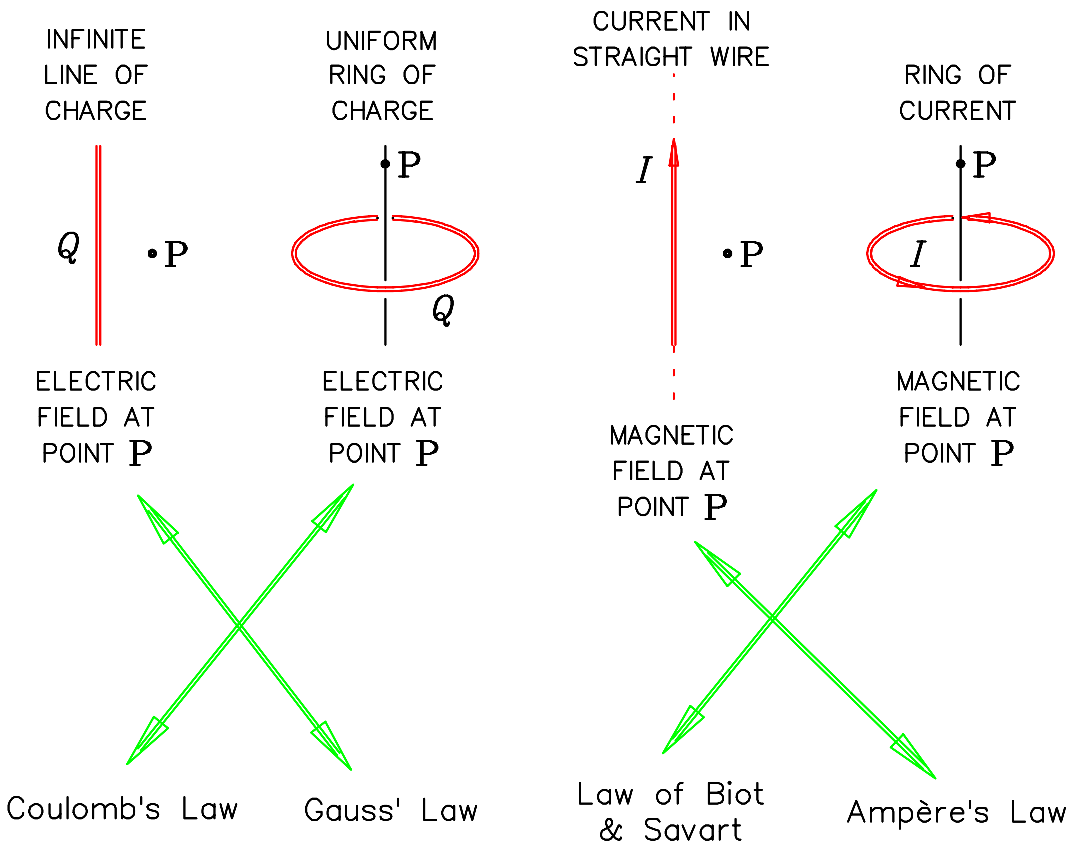


i. in the cavity $\boxed{\vec{\boldsymbol{E}}_{\text{cav}} = \dfrac{Q}{4\pi\epsilon_\circ}\left(\dfrac{\hat{\boldsymbol{r}}}{r^2}\right)}$
where $\vec{\boldsymbol{r}}$ is the position relative to the centre of the cavity. That is, just as if the conductor were completely removed.

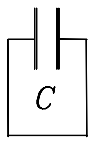
ii. in the conductor $\boxed{\text{Zero}}$ (as always).

iii. outside the big sphere $\boxed{\vec{\boldsymbol{E}}_{\text{out}} = \dfrac{Q}{4\pi\epsilon_\circ}\left(\dfrac{\hat{\boldsymbol{R}}}{R^2}\right)}$ where $\vec{\boldsymbol{R}}$ is the position relative to the centre of the big sphere. That is, just as if the charge were moved to the centre of the big sphere and then the conductor were completely removed.

(*d*) Shown below are four types of calculation problems and four "Laws" of Electricity and Magnetism. Match up (with connecting lines) each problem with the Law you should use to solve it with.

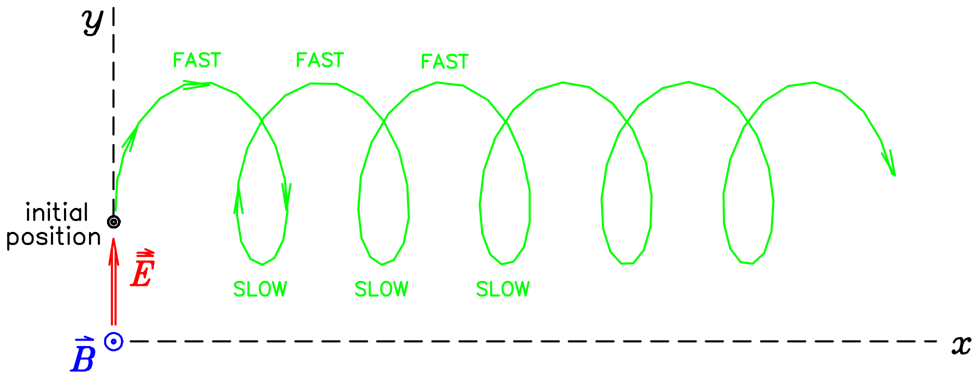


(*e*) The charge Q on any *shorted* capacitor will exhibit thermal fluctuations. If a 1 F capacitor is in thermal equilibrium at a temperature of 300 K, what is its *root mean square charge* $Q_{rms} = \sqrt{\langle Q^2 \rangle}$?



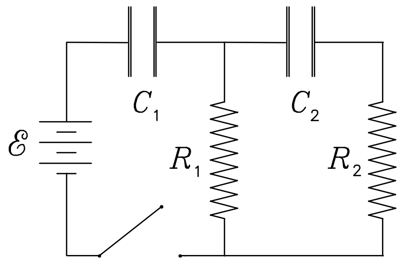
ANSWER: The energy stored in a charged capacitor is given by $E = \frac{1}{2}CV^2 = Q^2/2C$, so Q is a "degree of freedom" (like v_x) and has an associated mean thermal energy $\langle E \rangle = \langle Q^2 \rangle/2C = k_{\rm B}T/2$. Thus $\langle Q^2 \rangle = Ck_{\rm B}T$. With $C = 1$ F, $T = 300$ K and $k_{\rm B} = 1.3807 \times 10^{-23}$ J/K, this gives $\langle Q^2 \rangle = 0.4142 \times 10^{-20}$ and $\boxed{Q_{rms} = 0.6436 \times 10^{-10} \text{ Coulomb}}$.

(*f*) A proton is *initially at rest* in a uniform magnetic field out of the page. A uniform electric field is then applied in the $+y$ direction, perpendicular to the magnetic field. Describe the subsequent motion of the proton *qualitatively*, using a simple *sketch* and a few words. **ANSWER:** Initially the charge accelerates in the y direction (up); but as soon as it picks up some speed it starts to be bent to the right (x) by the magnetic field. Its direction is eventually reversed by this curvature and then it starts to slow down again until it goes around a bend again at the "slow end" and repeats the cycle. In the process it has drifted a bit to the right; this drift continues indefinitely. Some "drift chambers" use this method to collect the charge from ionization by elementary particles.



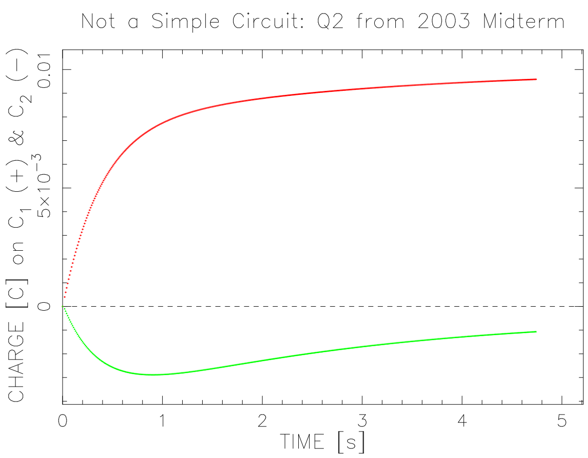
Does the proton ever move into or out of the page? *Explain.* $\boxed{\text{No.}}$
The initial velocity is zero and all accelerations are in the plane of the paper.

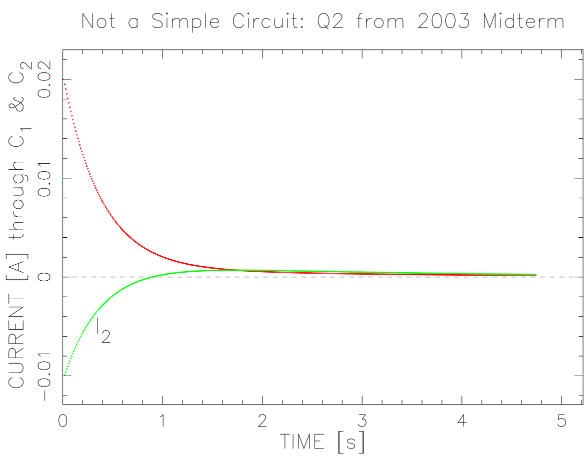
2. Not a Simple Circuit [20 marks]
Two identical 0.01 F capacitors and two identical 100 Ω resistors are used with a 1 V battery $\mathcal{E}$ to make the circuit shown. The capacitors are initially uncharged. *Describe what happens* after the switch is closed at $t = 0$. (You need not solve equations. Sketches are nice.)



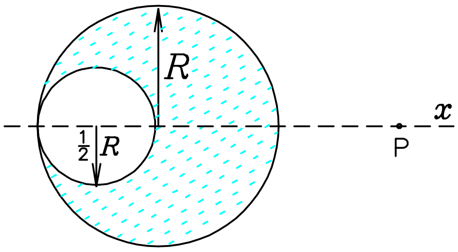
ANSWER: Initially, C_1 and C_2 are like "shorts" so the current flows equally through R_1 and R_2 in parallel. Both currents add to the charge buildup on C_1; so the charge on C_2 initially builds up half as fast as on C_1. Later, as C_1 approaches its "fully charged" state ($Q_1 \to C\mathcal{E}$) and the current decreases, C_2 reaches a state of "balance" and the charge Q_2 on C_2 goes through a maximum, after which it starts to discharge through the two resistors. (At that point the current in the right side reverses direction.) Eventually C_1 is fully charged to $C\mathcal{E}$ and acts like an "open circuit" while C_2 is completely discharged and all the currents die away. The time scale for these events is on the order of the time constant $RC = 1$ s.

For those who want to see the actual time dependences, here are some plots calculated numerically:





3. Composite Conductor [20 marks]
A long cylindrical conductor (a wire), viewed in cross-section at right, carries a uniform current density out of the page except in the off-centre cylindrical hole where the metal has been removed. The radius of the hole is half the radius R of the wire and the hole's central axis is located its $R/2$ to the left of the central axis of the wire, as shown. If $R = 1$ cm and the wire carries a net current of 1 A, calculate the resultant magnetic field at a position P on the x axis 1 cm to the right of the wire's right edge.



(a) [5 marks] Explain how you plan to solve this problem. **ANSWER:** Treat the wire as a superposition of an intact cylinder (without the hole) carrying a current I_0 plus a smaller, off-centre cylinder (the hole) carrying a current I_1 in the opposite direction. The resultant field is the superposition of the fields due to those two components.

(b) [15 marks] Do it. **ANSWER:** The area of the hole is 1/4 that of the whole wire, so the actual current I is only 3/4 of I_0, the current that the intact wire would be carrying at the same current density. Thus $I_0 = 4I/3$ and $I_1 = -I/3$. Both components produce a vertical (on this page) magnetic field at P, the first upward (in the same direction as the R arrow on the diagram) and the other downward. The two fields are given simply by Ampère's Law for a long cylindrically symmetric current distribution (outside the conductor):

$$B_0 = \frac{\mu_0 I_0}{2\pi} \cdot \frac{1}{(R+R)} = \frac{4}{3}\left[\frac{\mu_0 I}{2\pi R}\right]\left(\frac{1}{2}\right)$$

$$B_1 = \frac{\mu_0 I_1}{2\pi} \cdot \frac{1}{(R+R+\frac{R}{2})} = -\frac{1}{3}\left[\frac{\mu_0 I}{2\pi R}\right]\left(\frac{2}{5}\right)$$

$$B = \frac{1}{3}\left[\frac{\mu_0 I}{2\pi R}\right]\left(2 - \frac{2}{5}\right) = \frac{8}{15}\left[\frac{\mu_0 I}{2\pi R}\right] = \frac{8 \times 4\cancel{\pi} \times 10^{-7} \times 1}{15 \times 2\cancel{\pi} \times 10^{-2}}$$

giving $\boxed{B = 1.067 \times 10^{-5}\ \mathrm{T}}$. (The direction has already been specified.)

Comment: As announced at the beginning of the hour, this was a long and difficult exam. I did not expect anyone to successfully complete every part to every question; nor did I expect anyone to be bored! I hope everyone had a chance to at least look at the selection of questions and work first on those with which they felt most confident. Now that the exam is over, I encourage everyone to *go back and do the other questions*, the ones that looked scariest when you first saw them. I can think of no better way to boost your competence (and confidence) for the final exam, which is coming soon.... Meanwhile, I will be devising an algorithm for scaling the marks on this Midterm.