THE UNIVERSITY OF BRITISH COLUMBIA

Physics 122

Sessional Examination

SOLUTIONS

12 April 2002

TIME: $2\frac{1}{2}$ HOURS

INSTRUCTOR(S): Jess H. Brewer

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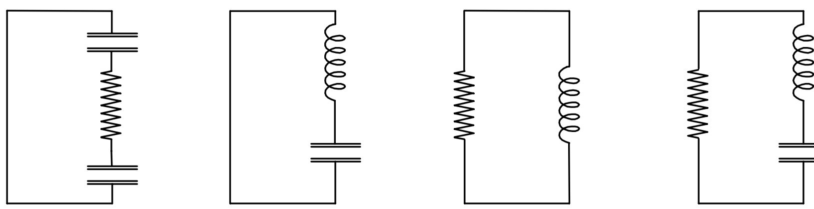
1. "QUICKIES" [40 marks — 5 each]

(*a*) What does "capacitance" have to do with "capacity"? Please answer *in words*. **ANSWER:** The *capacitance* of a capacitor is a measure of its *capacity* for *electric charge* — how much it will hold for a given applied voltage.

(*b*) A velocity selector has a magnetic field $\vec{B}$ perpendicular to an electric field of 50,000 V/m. We find that charged particles with velocity $v = 2.0 \times 10^5$ m/s perpendicular to both $\vec{E}$ and $\vec{B}$ pass through the device undeflected. What is the strength of $\vec{B}$ [in Tesla]? **ANSWER:** For this geometry, the LORENTZ FORCE LAW reads $F = q(E - vB)$. To get $F = 0$ we require $E = vB$ or $v = E/B$ or $B = E/v = 5 \times 10^4 / 2 \times 10^5$ or $\boxed{B = 0.25 \text{ T}}$.

(*c*) Suppose that you want to make a plastic camera lens with a *nonreflective coating* of a transparent material whose index of refraction is *higher* than that of the lens. *What is the optimum thickness* of the coating? *Explain.* **ANSWER:** The optimum thickness is "*much less than the wavelength of violet light.*" This will ensure that *all* visible light will be transmitted with minimal reflection, because the reflection off the first surface has a phase shift of π (reflection off a denser medium) but the reflection off the second surface does not; if there is a negligible path length difference, this implies destructive interference for the reflected light, regardless of wavelength. Thicker films work only for certain wavelengths. I want one of these lenses!

(*d*) Encircle any of the following circuits in which an initial *current* could possibly exhibit *simple exponential decay* without any oscillations.



ANSWER: All but the second circuit from the left. The rightmost circuit will exhibit simple exponential decay of the current if it is *overdamped.*

(*e*) To which of the laws represented by Maxwell's Equations did Maxwell actually make a direct contribution? *Explain briefly.* **ANSWER:** <u>AMPÈRE'S LAW</u>. It was incomplete (and incorrect) without the extra "DISPLACEMENT CURRENT" term introduced by Maxwell: $\frac{\partial}{\partial t} \iint_{\mathcal{S}} \vec{D} \cdot d\vec{A}$.

(*f*) A closed sphere of radius 5 cm is centred on the South-pole end of a 10 cm long bar magnet. The field strength at the pole tip is 0.1 Tesla. What is the net magnetic flux Φ_M *out of* the sphere? **ANSWER:** <u>Zero</u>. Since there are no magnetic monopoles, all "lines of $\vec{B}$" are *closed loops*. Therefore any flux that *leaves* a <u>closed</u> surface (regardless of its shape) must *come back in* somewhere else.

(*g*) Match up the names on the left and the principles on the right with the equations in the middle. (For each match, draw a connecting line.) **ANSWER:** This was meant as a gift!

Lorentz Force

Gauss' Law for Magnetostatics

Gauss' Law for Electrostatics

Ampère's Law

Faraday's Law

$$\oiint_{\mathcal{S}} \vec{\boldsymbol{D}} \cdot d\vec{\boldsymbol{A}} = Q_{\text{encl}}$$

$$\vec{\boldsymbol{F}} = q \left(\vec{\boldsymbol{E}} + \vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}} \right)$$

$$\oint_{\mathcal{C}} \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{\ell}} = -\frac{\partial}{\partial t} \iint_{\mathcal{S}} \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}}$$

$$\oiint_{\mathcal{S}} \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}} = 0$$

$$\oint_{\mathcal{C}} \vec{\boldsymbol{H}} \cdot d\vec{\boldsymbol{\ell}} = I_{\text{encl}} + \frac{\partial}{\partial t} \iint_{\mathcal{S}} \vec{\boldsymbol{D}} \cdot d\vec{\boldsymbol{A}}$$

A charged particle generally follows a curved path in a magnetic field.

"Lines of force" are continuous (unbroken) except where they begin and end on *charges*.

The "circulation" of magnetic field "lines" around an *open* surface is proportional to the net electric current flowing through that surface.

There are (apparently) no "magnetic monopoles."

Changing an electric field generates a magnetic field.

Changing a magnetic field generates an electric field.

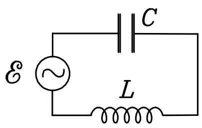
Moving charges generate magnetic fields.

Definitions: $\vec{\boldsymbol{D}} \equiv \epsilon \vec{\boldsymbol{E}}$ $\quad \epsilon \equiv \kappa \epsilon_0$ $\quad \kappa =$ dielectric constant.

$\vec{\boldsymbol{B}} \equiv \mu \vec{\boldsymbol{H}}$ $\quad \mu = \mu_0 (1 + \chi_m)$ $\quad \chi_m =$ magnetic susceptibility.

(In vacuum, $\kappa = 1$ and $\chi_m = 0$.)

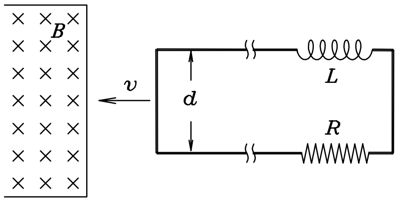
(*h*) In the AC circuit shown at right, $L = 0.01$ H, $C = 0.01$ F and the driving voltage is given by $\mathcal{E}(t) = \mathcal{E}_0 \cos(\omega t)$. *What is the one frequency ω which must never be used*, and *why?*



ANSWER: If $\omega = \omega_0 = 1/\sqrt{LC}$ or $\boxed{100\ \text{s}^{-1}}$ then the circuit is *in resonance* and any input power $\mathcal{E}_0$, no matter how small, will drive the circuit to destruction. A power-dissipating resistance is essential to avoid this.

2. HOT DUST [15 marks] Suppose that two large meteors made of pure carbon collide just outside the Earth's atmosphere and produce a huge number of "buckyballs" (C_{60} molecules) that then fall gently into the atmosphere and settle toward the ground. One buckyball has a mass of 1.1956×10^{-24} kg. After the buckyballs have had plenty of time to reach thermal equilibrium, assuming perfectly still air at 300 K, at what *altitude* will the concentration of buckyballs (per cubic meter of air) be exactly $1/e$ of their concentration at sea level? (Here e is the base of the natural logarithm.) **ANSWER:** The gravitational potential energy of a buckyball is $\varepsilon(h) = mgh$ where h is the altitude above sea level. If our "system" is just *the altitude of the buckyball*, then in thermal equilibrium the BOLTZMANN DISTRIBUTION implies that the probability of finding one buckyball at height h is proportional to $\exp(-\varepsilon/\tau)$ where $\tau \equiv k_B T$. Since all buckyballs share the same probability distribution, the density $D(h)$ of buckyballs has this same distribution. We don't need to know the normalization to find the *ratio* $D(h)/D(0) = \exp(-mgh/k_B T)$. This ratio will be e^{-1} when $mgh = k_B T$ or when $h = k_B T/mg$. Plugging in the numbers gives $\boxed{h = 353 \text{ m}}$.

3. MOVING CIRCUIT [15 marks]
The closed LR circuit shown at right moves to the left at a steady speed $v = 2$ m/s. Initially there is no current flowing in the circuit. At $t = 0$ the left end (width $d = 10$ cm) enters a region of uniform magnetic field $B = 1$ T into the page. Given: $L = 1$ H and $R = 4\ \Omega$.

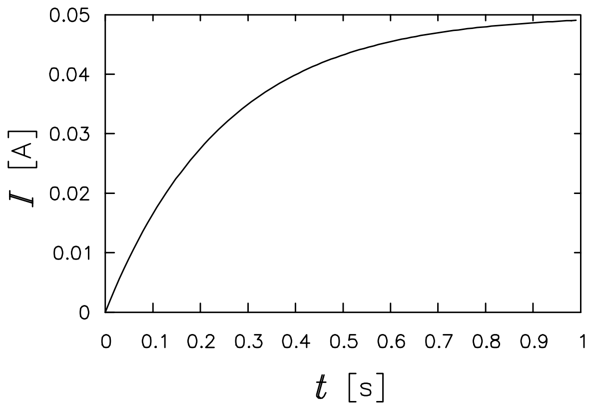


In the Figure, the ⊣⊢ symbols just represent "continuation" of a line that is too long to fit onto the page. Here they mean that the left part of the circuit is much longer than shown above: the circuit is 8 m long horizontally.

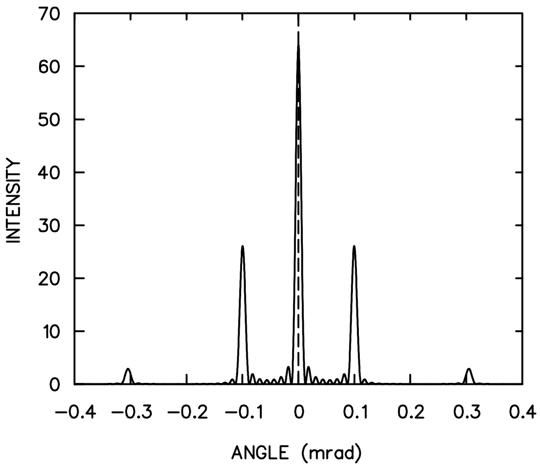
(*a*) [5 marks] Indicate on the Figure *which way current will flow* around the circuit for $t > 0$.
ANSWER: Once the circuit enters the field, Φ_M through the circuit starts *increasing* in the direction *into the page*. Therefore the induced EMF and the resultant current are in the direction that will produce a magnetic field *out of the page*, namely counterclockwise.

(*b*) [10 marks] *Sketch the current in the circuit as a function of time* from $t = 0$ to 1 s. Label both axes of your plot clearly and quantitatively.

ANSWER: For $t > 0$, $\mathcal{E} = -d\Phi_M/dt = Bvd$ or $\mathcal{E} = 0.2$ V. This constant EMF is like a *battery* forcing a current (initially zero) into the LR circuit. The subsequent time dependence should be familiar: $I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$ where $\tau = L/R$. In this case $\mathcal{E}/R = 0.2/4 = 0.05$ A and $\tau = 1/4$ s, so the plot looks like this:



4. GRATING [15 marks]



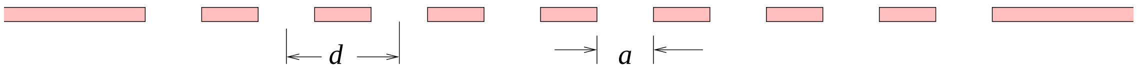
A grating is uniformly illuminated with infrared light of wavelength $\lambda = 1\ \mu$m incident normal to the plane of the grating, producing the interference pattern shown at left on a distant screen.

Make a detailed sketch of the grating itself, showing all dimensions.

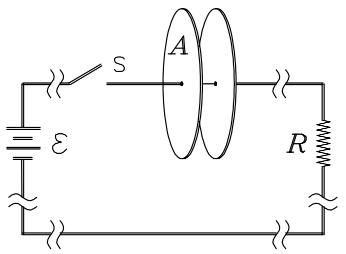
(Note: 1 mrad $= 10^{-3}$ radians.)

ANSWER: There are 6 secondary maxima between principal maxima, so there must be $\boxed{N = 8 \text{ slits}}$.

Let d be the distance between slits and let a be the width of one slit. The angle θ_1^I of the first *principal maximum* of the N-slit INTERFERENCE pattern is 0.1 mrad or $\theta_1^I = 10^{-4}$. This is a very small angle, so we may use the approximation $\lambda = d \sin\theta_1^I \approx d\theta_1^I$, giving $d = \lambda/\theta_1^I = 10^{-6}/10^{-4} = 0.01$ m or $\boxed{d = 10 \text{ mm}}$. The angle θ_1^D of the first *minimum* of the DIFFRACTION pattern is 0.2 mrad or $\theta_1^D = 2 \times 10^{-4}$. Again we can use the small-angle approximation $\lambda = a \sin\theta_1^D \approx a\theta_1^D$, giving $a = \lambda/\theta_1^D = 10^{-6}/2 \times 10^{-4} = 0.005$ m or $\boxed{a = 5 \text{ mm}}$. The entire grating looks something like this:



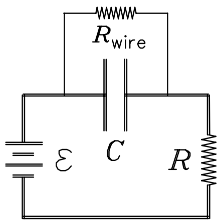
5. PARTIALLY SHORTED CAPACITOR [15 marks] — Read carefully!



A *partially shorted capacitor* is constructed from two parallel circular plates of area $A = \pi$ m^2 separated by a distance $d = \pi$ cm with a fine resistive wire ($R_{\text{wire}} = 100\ \Omega$) connecting the centre of one plate with the centre of the other. This device is placed in the circuit shown, with all other elements located far away. All other wires in the circuit have negligible resistance, except for the external resistor $R = 200\ \Omega$. The battery produces a potential of $\mathcal{E} = 3$ V. The capacitor is initially uncharged. At $t = 0$ the switch S is closed and current starts to flow.

(a) [2 marks] What is the *capacitance* of the capacitor? **ANSWER:** $\boxed{C = \dfrac{\epsilon_0 A}{d} = 0.8854 \times 10^{-9}\ \text{F}}$.

(b) [2 marks] After the switch has been closed for a long time ($t \to \infty$), what is the *current* flowing through the circuit? *Hint:* It may be helpful to *draw an equivalent circuit*.



ANSWER: The points of connection of the thin wire can be moved away from the centres of the plates to anywhere else on the same conductor without affecting the circuit behaviour (understanding topological equivalence is essential to circuit design), so what we have is a capacitor *in parallel* with a resistor on that leg of the circuit, as shown. Once the capacitor is fully charged, it "acts like an open circuit" and so at $t \to \infty$ *all the current flows through both resistors in series*, for an equivalent resistance of $R_{\text{eff}} = 300\ \Omega$ and therefore a current of $I_\infty = \mathcal{E}/R_{\text{eff}} = 3/300$ or $\boxed{I_\infty = 0.01\ \text{A}}$.

(c) [2 marks] What is the *charge* on the capacitor as $t \to \infty$? **ANSWER:** The voltage drop across the capacitor, Q/C, must be the same as the voltage drop across the wire, $I_\infty R_{\text{wire}} = Q_\infty/C$ so that $Q_\infty = C I_\infty R_{\text{wire}}$ or $\boxed{Q_\infty = 0.8854 \times 10^{-9}\ \text{C}}$.

(d) [2 marks] Immediately after the switch is closed, what is the *current* in the *fine resistive wire* connecting the two plates of the capacitor? *Hint:* It may be helpful to refer to the *equivalent circuit* you drew earlier. **ANSWER:** $\boxed{\text{Zero}}$. The uncharged capacitor initially "looks like a short circuit" and so there is no reason for any current to flow through R_{wire}.

(e) [2 marks] Immediately after the switch is closed, what is the *current* in the *external resistor*? **ANSWER:** The equivalent circuit is initially just the battery pushing a current I_0 through the external resistor R, so $I_0 = \mathcal{E}/R = 3/200$ or $\boxed{I_0 = 0.015\ \text{A}}$.

(f) [5 marks] Immediately after the switch is closed, what is the *magnetic* field $\vec{\boldsymbol{B}}$ (magnitude and direction) *inside* the capacitor as a function of the distance r away from the central axis? **ANSWER:** This is the hard part. There is (initially) no current in the thin wire, so the magnetic field is generated entirely by the DISPLACEMENT CURRENT: $\oint_{\mathcal{C}} \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 \dfrac{\partial}{\partial t} \iint_{\mathcal{S}} \epsilon_0 \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}}$ where the closed loop $\mathcal{C}$ is a circular path of radius r inside the capacitor (where by symmetry $\vec{\boldsymbol{B}}$ is everywhere parallel to the path and constant in magnitude) and $\mathcal{S}$ it the flat surface bounded by that loop (so that $\vec{\boldsymbol{E}}$ is uniform and normal to the surface) gives $(2\pi r)B = \mu_0 \epsilon_0 (\pi r^2)\, dE/dt$ or $B = \mu_0 \epsilon_0\, r\, dE/dt$. The direction of $\vec{\boldsymbol{B}}$ is out of the page at the top and into the page at the bottom. Now, the electric field between the plates is V/d where $V = Q/C$, so $\dfrac{dE}{dt} = \dfrac{1}{Cd}\dfrac{dQ}{dt} = \dfrac{I_0}{Cd}$. Thus $B(r) = \mu_0 I_0 \left(\dfrac{\epsilon_0}{Cd}\right) r$. You can plug in the numbers now or simplify further using $C = \dfrac{\epsilon_0 A}{d}$ to get $B(r) = \dfrac{\mu_0 I_0}{2A}\, r$. Either way, the result is $\boxed{B\ [\text{T}] = (3 \times 10^{-9})\ r\ [\text{m}]}$.