THE UNIVERSITY OF BRITISH COLUMBIA

Physics 122

SOLUTIONS

MIDTERM - 1 March 2002

TIME: 50 MINUTES

INSTRUCTOR(S): Jess H. Brewer

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1. QUICKIES [60 marks total]

(*a*) [12 marks] Encircle all the *true* completions of the sentence:

The temperature of an isolated system

i. always increases as the energy of the system increases.
Usually true, but not always. Consider the case of N spins in a magnetic field.

ii. always increases as the entropy of the system increases. Likewise.

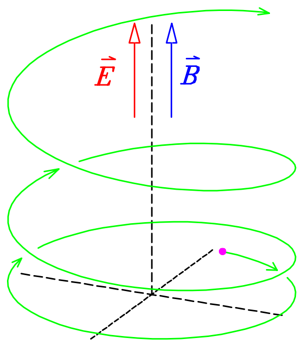
iii. always decreases as the entropy of the system increases. Usually not true.

iv. $\boxed{\text{doesn't depend on magnitude of either energy or entropy of system.}}$

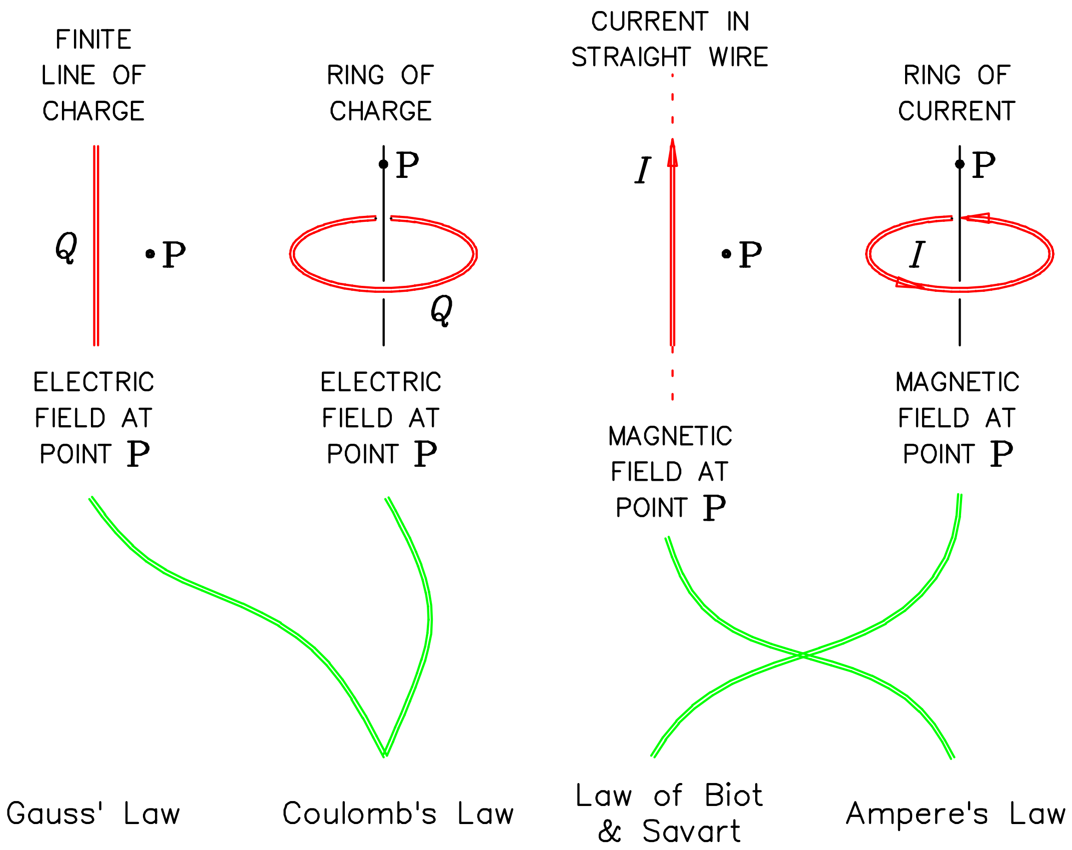
v. is higher if adding energy to the system increases its entropy more.
This applies to $1/T = dS/dU$, not to T.

vi. $\boxed{\text{is higher if adding energy to the system increases its entropy less.}}$

(*b*) [10 marks] A proton is initially moving perpendicular to a uniform magnetic field $\vec{\boldsymbol{B}} = B\hat{x}$. A uniform electric field $\vec{\boldsymbol{E}} = E\hat{x}$ is then applied parallel to the magnetic field. Briefly describe the subsequent motion of the proton in words and/or with a simple sketch. **ANSWER:** It will accelerate in the $\hat{x}$ direction (along the fields) while continuing to move in a circle in the y-z plane. Thus the path is a "stretched spiral" of constant radius and steadily increasing pitch.



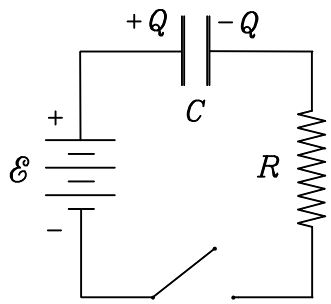
(c) [8 marks] Shown below are four types of calculation problems and four "Laws" of Electricity and Magnetism. Match up (with connecting lines) each problem with the Law you should use to solve it with.



(Gauss' Law cannot be used on a <u>finite</u> line of charge.)

(d) [10 marks] A cube 1 m on a side completely surrounds an electric dipole consisting of a positive electric charge of 1 C located 10 cm away from an equal-magnitude negative charge. What is the average value, taken over the whole surface of the cube, of the electric field normal to the surface? *Explain* your answer. **ANSWER:** Gauss' Law says that the integral of E_n, the normal component of the electric field, over any closed surface is proportional to the net charge within, in this case zero. The average E_n is the integral of E_n over the surface divided by the net surface area, so it is also $\boxed{\text{zero}}$.

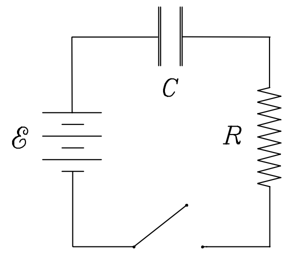
(e) [10 marks]
The capacitor in the circuit shown is initially charged to $Q = C\mathcal{E}$ where $\mathcal{E}$ is the voltage on the battery. Describe in detail what happens when the switch is closed.



ANSWER: $\boxed{\text{Nothing happens}}$.

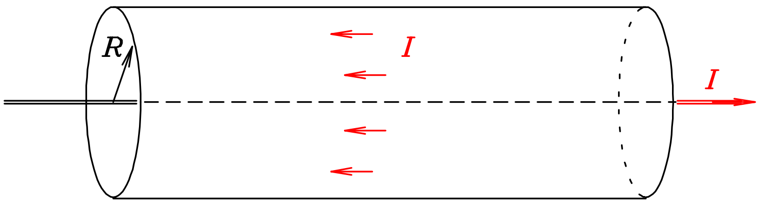
The capacitor is already fully charged.

(f) [10 marks]
The initially *un*charged capacitor ($C = 1$ F) is in series with a battery whose voltage is $\mathcal{E} = 10$ V and a resistor ($R = 100\ \Omega$). Describe in detail what happens when the switch is closed. **ANSWER:** Initially, the uncharged capacitor looks like a short; a current $I = \mathcal{E}/R$ flows through the resistor as if C weren't there.



Then, as charge builds up on C, it "pushes back" more and more, slowing down I until the charge on the capacitor approaches $Q_f = C\mathcal{E}$ asymptotically. The time constant is $RC = 100$ s. If you like formulae, $I(t) = (\mathcal{E}/R)\exp(-t/RC)$ and $Q(t) = C\mathcal{E}[1 - \exp(-t/RC)]$. I won't bother to make sketches, but they are just as good as formulae; better, really.

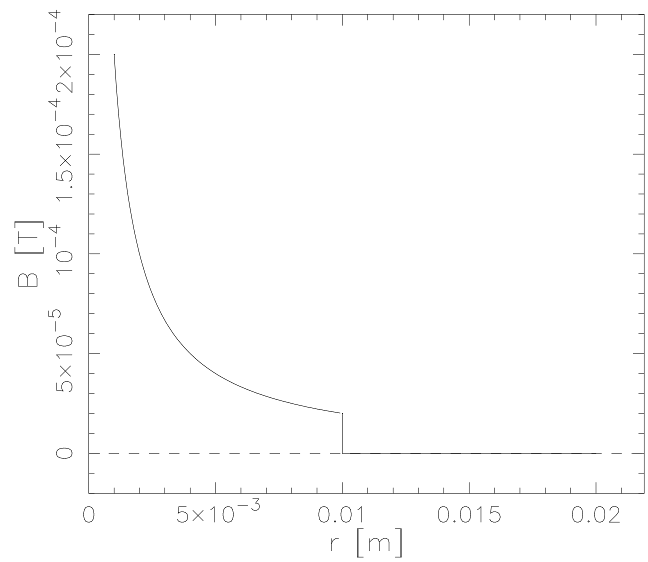
2. COAXIAL CABLE [20 marks] A net current of $I = 1$ A flows down a long, thin central wire and back along a long, thin-walled cylindrical outer conductor of radius $R = 1$ cm, as shown. The returning current density is uniformly distributed over the outer conducting shell.



(a) [5 marks] In what *direction* is the vector magnetic field $\vec{\boldsymbol{B}}$ in various regions? (Indicate on the sketch and/or in words.)
ANSWER: By symmetry, the magnetic field lines must always be circles perpendicular to the axis of the cylinder and centred on the inner wire. Inside the shell the field points in the direction of the fingers of your right hand when your thumb points along the current on the wire. Outside the shell the field is zero, because the total current through any Ampèrian loop around the outside is zero.

(b) [10 marks] Find a complete expression for the magnetic field strength B as a function of r (the distance from the central axis), I and R.
ANSWER: As mentioned above, Ampère's Law gives $\boxed{B = 0 \text{ for } r > R}$ because there is the same current flowing in both directions through an Ampèrian loop outside the outer shell. Inside the shell, the "return" current has no effect because it does not "link the loop", so we get the same answer as if there were just the inner wire: $2\pi r B = \mu_\circ I$ or $\boxed{B(r < R) = \mu_\circ I/2\pi r}$. A lot of people turned in nice solutions for more complicated problems, presumably because they didn't read the question carefully and/or because of the "hammer/nail syndrome" (the homework problem being the "hammer" you have learned to use, every other Ampère's Law problem then looking like a "nail").

(c) [5 marks] *Plot* $B(r)$ from $r = 0$ to $r = 2R$, labelling your axes clearly (including the vertical scale in Tesla and the horizontal scale in metres).



3. HOT CHARGE [20 marks] A capacitor is made from two parallel metal plates 10 cm apart. Each plate is a square 1 m on a side. There is a charge of $+Q$ on one plate and $-Q$ on the other plate, where $Q = 10^{-11}$ C. A tiny glass bead containing a *single excess electron* (which we will assume is "stuck" in the bead) is free to move between the plates; let x be its distance away from the positive plate. The bead is in thermal equilibrium with the surrounding air at $T = 300$ K. Assume that the air is perfectly still (no air currents) and neglect any effects of gravity.

   (a) [5 marks] What is the *potential energy of the bead* as a function of x, if we define its potential energy to be zero at $x = 0$? **ANSWER:** There is a uniform electric field $E = \sigma/\epsilon_\circ = Q/A\epsilon_\circ = 10^{-11}/(1 \times 0.885 \times 10^{-11}) = 1.13$ V/m between the plates, so the force on a charge $-e$ is $F = Ee = 1.13 \times 1.602 \times 10^{-19} = 1.81 \times 10^{-19}$ N pulling it toward the positively charged plate. This is analogous to the force of gravity at the Earth's surface. To "lift" the bead a "height" x "above" the positive plate, takes Fx worth of work and raises the potential energy of the bead by $\varepsilon = Fx$. This is the answer. $\boxed{\varepsilon = Eex}$ or ε [J] $= 1.81 \times 10^{-19}\ x$ [m].

   (b) [5 marks] If you were to take into account the small but finite *dielectric constant* of the *air* between the capacitor plates, would the potential energy of the bead get *larger* or *smaller* at the same nonzero x position? *Explain.* (No credit for a guess.) **ANSWER:** A dielectric increases the capacitance of a capacitor, which then requires less applied voltage to store the same charge; turning this around, with a given charge on it the capacitor will have a smaller voltage drop across it (and therefore a smaller electric field inside, since the plate separation has not changed) and therefore a weaker force on the charged bead. The bead's potential energy will therefore be slightly $\boxed{\text{smaller}}$ due to the air. Very slightly.

(*c*) [10 marks] What is the *ratio* of the probability of finding the bead exactly halfway between the plates ($x = 5$ cm) to the probability of finding it touching the positive plate ($x = 0$)? **ANSWER:** Boltzmann says the probability of finding a microsystem in a state of energy ε is proportional to $\exp(-\varepsilon/k_{\rm B}T)$ if the system is in thermal equilibrium with a big heat reservoir at temperature T. This is such a situation, where the microsystem in question is just the bead's distance from the positive plate. So $\mathcal{P}(x) = c\,\exp(-\varepsilon/k_{\rm B})$ where we don't know the normalizing constant c. But we don't need to know it to calculate a *ratio* of probabilities: $\dfrac{\mathcal{P}(x = 0.05\,\mathrm{m})}{\mathcal{P}(x = 0)} = e^{-Fx/k_{\rm B}T} = \exp\left(-\dfrac{1.81 \times 10^{-19} \times 0.05}{1.38 \times 10^{-23} \times 300}\right) = e^{-2.18}$ or $\boxed{\text{ratio} = 0.113}$.

— *FINIS* —