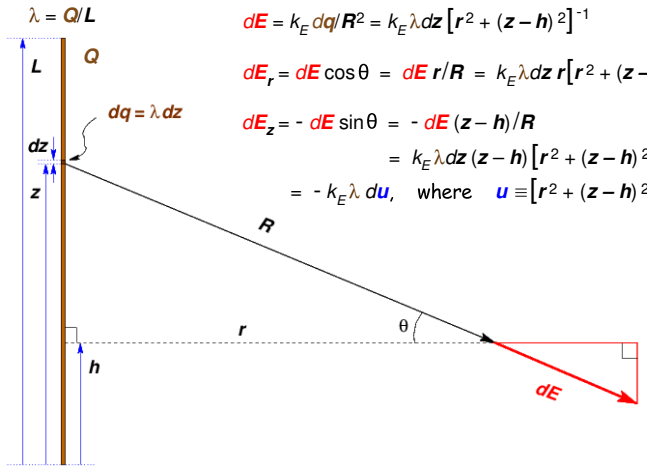
Finite Rod of Charge – general case



$dE = k_E\, dq/R^2 = k_E \lambda dz \left[r^2 + (z-h)^2\right]^{-1}$

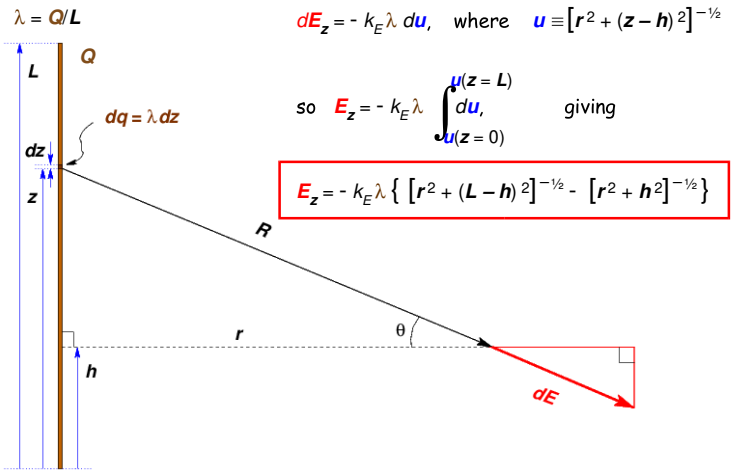
$dE_r = dE \cos\theta = dE\, r/R = k_E \lambda dz\, r\left[r^2 + (z-h)^2\right]^{-3/2}$

$dE_z = -\, dE \sin\theta = -\, dE\,(z-h)/R$

$= k_E \lambda dz\,(z-h)\left[r^2 + (z-h)^2\right]^{-3/2}$

$= -\, k_E \lambda\, du, \quad \text{where} \quad u \equiv \left[r^2 + (z-h)^2\right]^{-1/2}$

Finite Rod of Charge – general case

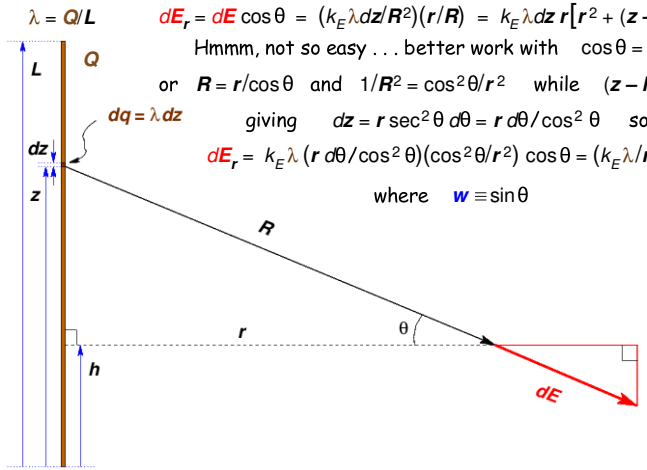


$dE_z = -\, k_E \lambda\, du, \quad \text{where} \quad u \equiv \left[r^2 + (z-h)^2\right]^{-1/2}$

so $E_z = -\, k_E \lambda \int_{u(z=0)}^{u(z=L)} du,$ giving

$$E_z = -\, k_E \lambda \left\{ \left[r^2 + (L-h)^2\right]^{-1/2} - \left[r^2 + h^2\right]^{-1/2} \right\}$$

Finite Rod of Charge – general case



$dE_r = dE \cos\theta = (k_E \lambda dz/R^2)(r/R) = k_E \lambda dz\, r\left[r^2 + (z-h)^2\right]^{-3/2}$

Hmmm, not so easy . . . better work with $\cos\theta = r/R$

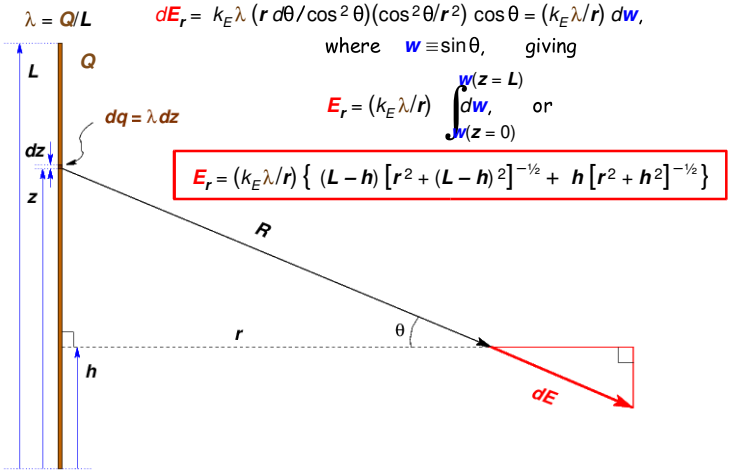
or $R = r/\cos\theta$ and $1/R^2 = \cos^2\theta/r^2$ while $(z-h) = r\tan\theta,$

giving $dz = r\sec^2\theta\, d\theta = r\, d\theta/\cos^2\theta$ so that

$dE_r = k_E \lambda\,(r\, d\theta/\cos^2\theta)(\cos^2\theta/r^2)\cos\theta = (k_E\lambda/r)\, dw,$

where $w \equiv \sin\theta$

Finite Rod of Charge – general case



$dE_r = k_E \lambda\,(r\, d\theta/\cos^2\theta)(\cos^2\theta/r^2)\cos\theta = (k_E\lambda/r)\, dw,$

where $w \equiv \sin\theta,$ giving

$E_r = (k_E\lambda/r) \int_{w(z=0)}^{w(z=L)} dw,$ or

$$E_r = (k_E\lambda/r)\left\{ (L-h)\left[r^2 + (L-h)^2\right]^{-1/2} + h\left[r^2 + h^2\right]^{-1/2} \right\}$$