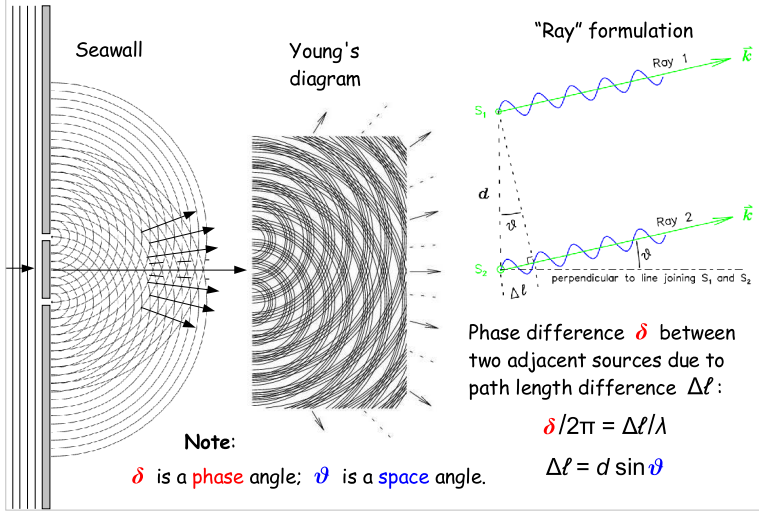
Two-Slit Interference



Phase difference δ between two adjacent sources due to path length difference $\Delta\ell$:

$\delta/2\pi = \Delta\ell/\lambda$

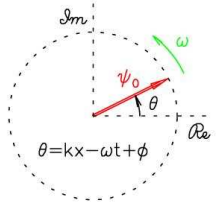
$\Delta\ell = d \sin\vartheta$

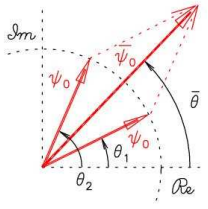
Note:

δ is a phase angle; ϑ is a space angle.

Phasors

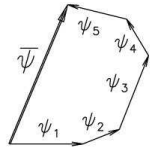
The amplitude of a single wave at a given point can be pictured as the Real part of a vector rotating at ω in the complex amplitude plane ("phase space"):



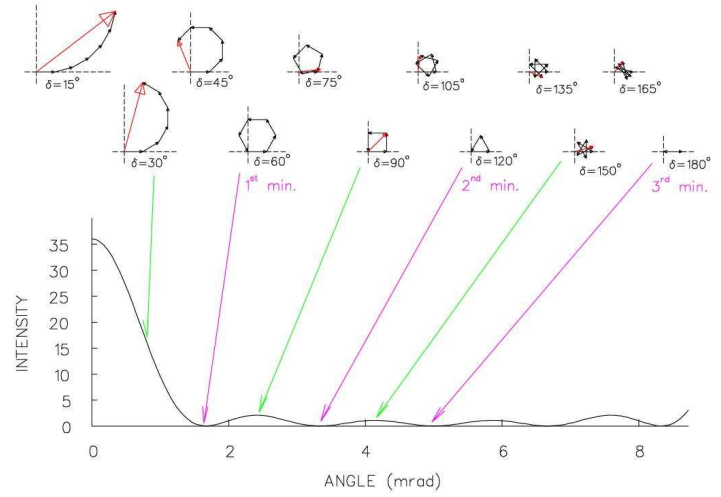


Two such amplitudes (for two waves with the same ω but "out of phase" at that point) can be "added tip to tail" to get a resultant vector whose real part is the net amplitude there.

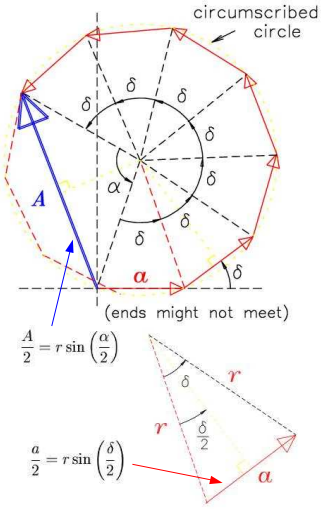
This works equally well for a larger number of such "phasors" with different lengths, as long as they all have the same ω.



6-Slit Interference



N-Slit Interference



Trigonometry on the phasor diagram gives

$$\frac{A}{2} = r\sin\left(\frac{\alpha}{2}\right) \qquad \frac{a}{2} = r\sin\left(\frac{\delta}{2}\right)$$

$$\alpha = 2\pi - N\delta$$

$$A = a\left[\frac{\sin\left(N\frac{\delta}{2}\right)}{\sin\left(\frac{\delta}{2}\right)}\right]$$

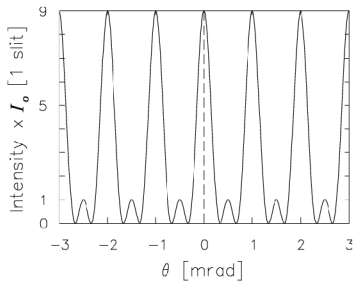
where

$$\delta = 2\pi\left(\frac{d}{\lambda}\right)\sin\vartheta$$

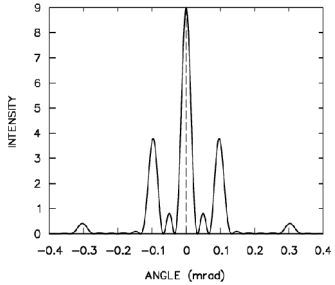
The interference pattern for N equally spaced slits exhibits $(N-1)$ *minima* and $(N-2)$ *secondary maxima* between each pair of principal maxima.

N-Slit Interference Patterns

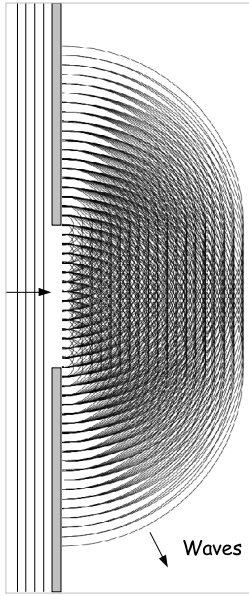
Ideal 3-slit intensity pattern



3-slit Grating intensity pattern



What's the difference? Gratings have **finite-width slits!**



Waves coming through 16 small gaps in a seawall:

Begins to look more like **one big gap**....

(This is what we call "**diffraction**".)

"Central Maximum"

Suppose we divide up one wide (a) slit into N equal "pseudoslits" of width a/N and let $N \to \infty$?

Waves bend around corners !

Diffraction as "∞-Slit Interference"

$$A = a\left[\frac{\sin\left(N\frac{\delta}{2}\right)}{\sin\left(\frac{\delta}{2}\right)}\right]$$ where $$\delta = 2\pi\left(\frac{d}{\lambda}\right)\sin\vartheta$$ and a = amplitude from 1 slit $= A_0/N$ *where* A_0 *is* $\sqrt{(I_0)}$.

Now let a represent the width of the big slit (sorry for the change of notation!) and replace d with a/N, giving $\delta = 2\pi\,(d\sin\vartheta)/\lambda = 2\pi\,(a/N)(\sin\vartheta)/\lambda$

so $N\delta/2 = \pi\,(a\sin\vartheta)/\lambda \equiv \alpha$ (a new α, sorry!) and $\delta/2 = \alpha/N$.

Thus $A = (A_0/N)\,(\sin\alpha)/\sin(\alpha/N)$. Now we let $N \to \infty$ and use the small angle approximation $\sin(\alpha/N) \approx (\alpha/N)$ to get $A = A_0\,(\sin\alpha)/\alpha$ and therefore

$$I = I_0\,[(\sin\alpha)/\alpha]^2$$

which has its first minimum where $\alpha = \pi$ or $a\sin\vartheta_1 = \lambda$.