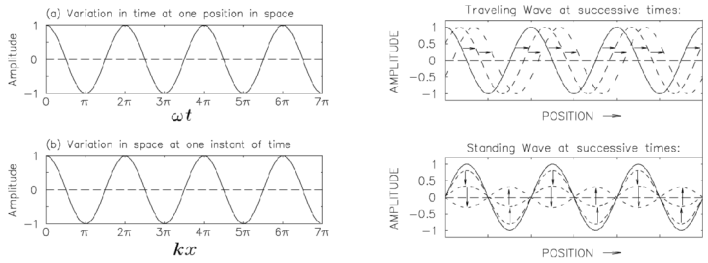
WAVES



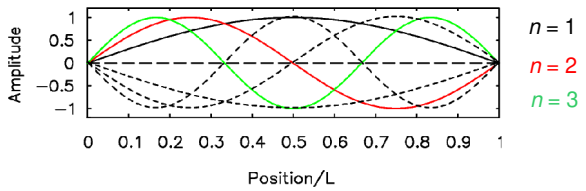
$\omega = 2\pi/T \qquad k = 2\pi/\lambda$

$\omega/k = \lambda\,\nu = c$

MODES

of

Standing Waves



If the ends of a string are held fixed, the only allowed **modes** of vibration are those with an integer number of half-wavelengths in L: $L = n\,(\lambda_n/2)$ or

$$\lambda_n = 2L/n$$

De Broglie's Hypothesis

Planck and (in 1905) Einstein had explained that waves (light) propagate as quanta ("photons") of energy $\epsilon = h\,\nu$ where ν is the frequency of the wave's oscillation and $h = 6.626 \cdot 10^{-34}$ J-s is Planck's constant.
It was also well known that the energy of such a wave is equal to its momentum times its speed of propagation: $\epsilon = p\,c$. For any wave, $c = \lambda\,\nu$, and so, for photons, $p = h/\lambda$.

Louis de Broglie simply postulated that if light acts like particles, maybe electrons (or any particles) also behave like waves, and in particular have wavelengths given by

$$\lambda = h/p.$$

Bead on a Frictionless String

$$\lambda_n = 2L/n$$

$$\lambda = h/p$$

$$p_n = h/\lambda_n = nh/2L$$

Nonrelativistically, $p = m\,v = m\,2L/\Delta t$ so $\Delta t_n = 2Lm/p_n = 4L^2m/nh$

giving $E_n = p_n^2/2m = n^2\,h^2/8m\,L^2$

and $F_n = 2p_n/\Delta t = n^2\,h^2/4m\,L^3$

Relativistically...

Total Relativistic Energy: $E_n^2 = p_n^2 c^2 + m^2 c^4$ (Always true!)

For $E_n \gg m c^2$ ("ultrarelativistic") we have $E_n \approx p_n c$ and if we take $m_{eff} = E/c^2$ and $n = 1$, then $m_{eff} \approx h/2Lc$.

An object in a circular orbit around a mass M has an orbital velocity $v_{\rm orb} = \sqrt{\frac{GM}{R}}$. If $v_{\rm orb}$ = c, then even light can't escape beyond the Schwarzschild radius, $R_S = \frac{GM}{c^2}$. What does it take to get $R_S = L$?

$$L = \frac{Gm_{\rm eff}}{c^2} = \frac{Gh}{2Lc^3} \quad \text{or} \quad \boxed{L_{\rm Planck} = \sqrt{\frac{Gh}{2c^3}}}.$$

If even a photon is confined to this small a region, the effect of the confinement will raise its energy enough to make it a **black hole**!

(Numbers are off by small factors like 2 or π.)