GENERAL RELATIVITY & COSMOLOGY

As Elementary Particle Physicists direct their attention "down" toward the indescribably tiny, so Cosmologists turn their gaze "upward" toward the unfathomably huge. Of course, these days both are increasingly likely to be incarnate in the same individual - I'll get to that later. As one who has never looked through a telescope larger than I could carry, I am certain to give short shrift to the magnificent observational science of **astronomy**, which provides **cosmology** (a theoretical discipline) with all its data. But a summary of the former without good colour plates of star fields and nebulae would be a terrible waste anyway, so I hope I have motivated the curious to go out and read a good Astronomy book on their own. Moreover, I am so ignorant of General Relativity and most of the fine points of Cosmology that I really have no business writing about either. Therefore I must content myself with a justification in terms of my "unique point of view," whereby I excuse the following distortions.

GR.1 Astronomy

Having just declared my intention *not* to cover **astronomy**, here I start right in with it! Well, I want to make a few abstract generalizations about the subject. The first is a commentary on the idea of an *observational* science in a Quantum Mechanical *millieu*. Until recently, all astronomical observations were made by detecting *light* emitted by distant objects a long time ago. Nowadays Astronomers detect the full range of the electromagnetic spectrum, from long-wavelength radio waves to gamma rays, as well as the odd *neutrino*,[1] but the qualitative picture hasn't changed: a virtual quantum is emitted at a distant source and absorbed here on Earth; by measuring the relative intensity of such quanta arriving from different directions, we get a picture (literally) of the Universe around us. On the one hand, we cannot detect the photons without annihilating them; in this sense the act of **measurement** interferes with the system being measured, as Quantum Mechanics has taught us to expect. On the other hand, it is reasonable to expect that our interference is only with the photons themselves, not with their distant emitters; and in this sense the Astronomer is an awfully good approximation to the classical **observer**.

The next philosophical point is that the photons we detect on Earth may have been "in transit" for millions or even billions of years, depending upon how far away their source was when they were emitted. Thus as we look *outward* to the distant galaxies we are also looking *backward* in time. Sort of. So if we see the same sort of **spectrum** (including, for instance, the ubiquitous hydrogen atom emission lines) from a star in another galaxy as we do from Sol, it means that the "Laws of Physics" are pretty much the same here and now as they were there and then. This gives a comforting sense of stability and permanence, even if our individual destinies are short and unknown.

In recent decades humans have developed the technical ability to *go and have a closer look* at other bodies in our own Solar System; this is absolutely delightful and has rekindled interest in Astronomy among the people who end up paying for it, better yet! However, it probably will come to be known by a different name (*e.g.* **planetology**) simply because of the increased scope of the Experimenter's capacity to interfere with the Observed. Ultimately, humans will again set foot on other worlds [as we did back in 1969 and the early 1970's - doesn't anyone remember?!] and carry the Laboratory to the

[1] The Sudbury Neutrino Observatory (SNO), now under construction in a Canadian mine shaft, will revolutionize this technology; nevertheless, the best one can hope for is some rough estimate of the direction of the source of individual neutrinos. The pesky critters just don't interact much! (Which is why they get here at all!)

stars where whole new categories of information can be gathered. However, the sheer distance of other stars makes patience a virtue in such plans....

GR.1.1 Tricks of the Trade

Since Astronomers began to chart the heavens (probably before recorded history as we know it), they have been learning *tricks* for finding out more about the stars than would seem possible, given their limited experimental tools. I don't know many of these, but I can point out a few of the important ones.

Parallax

When you watch a distant object out of the corner of your eye, you can keep it in view without turning your head even though you walk some distance at the same time, as long as you walk in a straight line. However, if the object is about the same distance away as the length of your walk, you will end up looking over your shoulder if you insist on keeping an eye on it. This is the essence of **parallax**, the shift of the apparent direction of a source as the observer changes position - which might not seem to be much help to Astronomers, until you realize that *the Earth moves* quite some distance every year in its path about Sol. By carefully measuring the angular shift in a star's position throughout a year, Astronomers can gauge its distance from the Earth out to an impressive range.

Spectroscopy

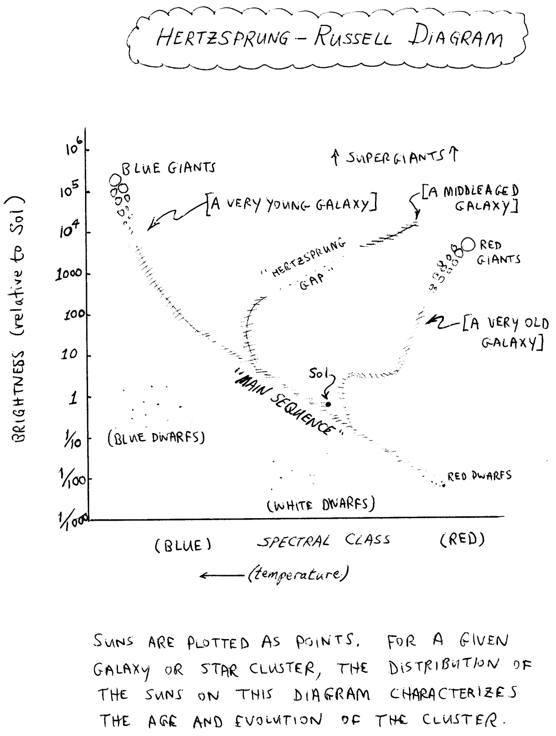
Meanwhile, looking at the **spectrum** of light from a star can tell us (a) how *hot* it is [recall the **blackbody** spectrum] and (b) what sort of *atoms* are in its "chromosphere" [the hot surface that we see]. Finally, the sheer *brightness* of the star (combined with a knowledge of its *distance* and *temperature*) tells us how *big* it is.

GR.1.2 Astrophysics

Putting together lots of such information has allowed a large number of stars to be *catalogued*, with the result that certain combinations of brightness and spectral "signatures" can be generally assigned to stars of a given age, size and character even before their distance is known empirically by **parallax** measurements. In this way a great deal has been learned about **stellar evolution** and (by inference) about the nuclear reactions in the cores of stars. This is the science of **astrophysics**, which differs from **astronomy** in that the latter seeks mainly to *observe* while the former seeks to *explain* the stars.

Astrophysical theories of stellar evolution are wondrous detailed, which suggests that I omit further attempts to describe them here. It is important to note, however, that much of the edifice of **cosmology** rests upon the internal consistency and predictive power of these theories.

Figure 1 A cartoon version of the **Hertzsprung-Russell diagram** showing the common categories of stars arranged by their **spectral class** (colour) and **brightness**.



GR.2 Bang!

As spectroscopists began to study more and more distant stars, they noticed a peculiar effect: the Doppler effect for light from distant stars [apparent in the H atom line spectrum, for instance] was not randomly scattered between red and blue shifts, as might be expected for a Universe full of stars "milling about." Instead, Hubble discovered that *the more distant the star, the bigger the* **red shift**. That is, all the other stars are, on average, moving *away from us* ; and *the more distant the star, the faster it is receding.*

It was a relatively easy matter to estimate from **Hubble's constant** how far away a star would have to be in order to be receding from us at the speed of light; the answer was in the neighbourhood of 10-20 billion light years. Since none can be moving any *faster* than the speed of light, this sets a crude limit on the *size of the Universe.*

Moreover, if this has been going on for 10-20 billion years, then all those stars and galaxies are *shrapnel* from an *explosion* 10-20 billion years ago that sent us all flying apart at velocities up to the speed of light. This scenario is known as the **Big Bang** model of the origin (and subsequent evolution) of the Universe.

What a picture! In the moment of Creation, all the matter in the Universe was at a single point, after which [to use the refined understatement of Cosmologists] "it began to expand." Initially the energy density was rather high, obviating all our notions about **elementary particles**, the heaviest of which looks like empty space by comparison. Only after the Universe had expanded and cooled by many, many orders of magnitude was it possible for the particles we know to "freeze out" and begin to go their separate ways.

Modern Cosmologists spend a great deal of their time worrying about the details of the "Early Universe," meaning the period from "$t = 0$" of the **Big Bang** until today's elementary particles condensed from the primal fireball. This explains (as promised) why there is often not much separation between **cosmology** and **elementary particle physics** - basically, the *big* was once *small*.

GR.2.1 Crunch?

This raises the question: Will it be small again someday? Is the present trajectory of matter in the Universe an "escape trajectory" so that the Universe will keep on expanding indefinitely, or is there enough mass present to *bind* the Universe - slowing down the "shrapnel" by gravitational attraction until it stops and begins to fall inexorably inwards. . . ?

To the best of my knowledge (which isn't all that impressive), opinion is divided. No one has been able to account for enough mass to keep the Universe "closed" (bound), so that it looks like a "**Big Crunch**" is not in store for us. On the other hand, a careful analysis of the present distribution of matter in the Universe suggests (or so I am told) that a "wide open" Universe (forever expanding) is not compatible with its present homogeneity. In fact, the theorists would be happiest with a perfect balance so that the Universe can't quite make up its mind whether it is bound or not! [This would appeal to anyone, but I think they actually have arguments why it must be so.] If this is the case, we must be missing two things: (1) a lot of mass that doesn't interact much and hence is known as **dark matter** composed of Weakly Interacting Massive Particles or **wimp**s; (2) any idea of the mechanism that ensures such incredibly "fine tuning" of the so-called **cosmological constant** - if it were infinitesimally *larger*, the Universe would have collapsed back upon itself in a matter of seconds, while a slightly *smaller* value would have us lost in empty space by now.

Am I out of my depth here, or what?

GR.3 Cosmology and Special Relativity

So far I have been sweeping the worst confusion under the rug.

First off, when we talk about "the Universe today," we mean "what we see today." This isn't quite fair, since the light we detect from distant objects was emitted a *long* time ago, maybe almost at the beginning of time! We have no way of knowing, even in principle, what those objects have been up to since then. Maybe they are all gone by now.

This creates a problem with *energy conservation* : since every star is in a different inertial reference frame from every other, what is *simultaneous* for one is not for another; in that case, how does one talk about *energy conservation* on a Cosmic scale? *When* do the books get balanced, according to *whose* perspective? I don't know of any resolution for this confusion. Perhaps energy conservation is an obsolete concept on the *large scale*.

GR.3.1 I am the Centre of the Universe!

On the other hand, the **Big Bang** picture does make it possible to resolve an old conflict between Ergocentric and Heliocentric Cosmologies. All the "bits of shrapnel" were once in the same place and have been flying apart ever since; in the crude approximation that their trajectories are non-interacting (*i.e.* disregarding the little deflections caused by gravitational attractions between neighbours), each one is perfectly justified in regarding itself as *at rest* while the others are all in motion. If I am at rest now, then (in this approximation) I have been at rest all along, and am still at the centre of the Universe where the **Big Bang** took place, whereas all you other bits are flying off to infinity.

Even if you insist upon a *geometrical* definition of the "centre of the Universe," I am still at its centre, for what can we possibly mean by the geometrical centre but the point equidistant from all the most rapidly receding bits - namely, photons and other massless particles moving at the speed of light. Since these were all emitted initially from the same point where I was then, and all are moving away in every direction at the same speed (guaranteed by the $\mathcal{STR}$), this is still the centre.

Of course, every other fragment is equally entitled to the same point of view - we are *all* at the centre of the Universe, as viewed in our own reference frame![2]

GR.4 Gravity

Cosmology is intimately involved with **gravity**, about which we may have a lot of instincts but not much accurate knowledge. Here's where we finally get down to the hard part. The first trick is to understand the only interaction that really matters in today's Universe: **gravity**. To do it right, of course, we must formulate a *relativistic* theory, since all those distant stars are moving away from us at velocities approaching the speed of light. Enter Albert Einstein, again.

GR.4.1 Einstein Again

Encouraged by his successes with Special Relativity and Quantum Mechanics, Albert tackled the thorny problem of **General Relativity** (the behaviour of Physics in *accelerated reference frames*) with his characteristic *élan*. The first difficulty was in distinguishing between *truly* accelerated frames (like a compartment in a rocket) and frames that only *seem* to be accelerated (like where you are sitting). Consider: if you can't look out the window, how do you *know* you are being pressed into the seat of your chair by the Earth's *gravity*, as opposed to being in a rocket somewhere in interstellar space accelerating "up" at 9.81 m/s^2? Well, yes, you walked into the room from outside and sat down just a short while ago; but suppose you had lost your memory? How can you *tell* (by experiment) which is the case?

[2]Once again Physics comes around to the same conclusion that has been reached by Psychology.

The Equivalence Principle

Einstein, following his usual æsthetics of simplicity, assumed the "dilemma" was its own solution - namely, *you can't* tell an accelerated reference frame from a reference frame in a gravitational field. This is known as the **equivalence principle**:

> No experiment performed in a closed system can tell whether it is in an *accelerated* reference frame or a reference frame in a *gravitational field*.

If you wake up in a closed box and you experience "weight" (as one normally does on Earth), *there is no way* to be sure you are actually being attracted by gravity, as opposed to being in a spaceship (far from any stars or planets) which is accelerating at one "gee." What's more, if Einstein is right, *no matter how clever you are* you will not be able to measure *any* phenomenon from which you can tell the difference. The two cases are *perfectly equivalent*, hence the name of the Principle.[3]

So far this Principle agrees with experiments, which has led people to look for ways to make the statement, "A gravitational field is *the same thing* as an accelerated reference frame," sound reasonable. To make any progress along these lines we have to turn to an analysis of our notion of "acceleration" - *i.e.* of the nature of space and time, and therefore of *geometry*.

GR.4.2 What is Straight?

If we want to do **geometry**, the first thing we need is a *straightedge*. Any straight line will do. What shall we use? Well, modern surveyors are mighty fond of *lasers* for the simple reason that *light travels in a straight line*. (If light doesn't, what does?!) At least in empty space this must be true. So if we like we can *define* a "straight line" in 3-space (x, y, z) to be the path of a ray of light. We call this path a **geodesic** of space for an important reason that is best explained by analogy [like most topics in Relativity].

Consider *air travel* on Earth. Most intercontinental flights take routes called "great circles" which may go over the North Pole *etc.* This is because these are *the shortest paths between two points* on the Earth, subject to the *constraint* that one must travel essentially in *two dimensions* along the *surface* of the Earth. Such lines, the shortest distances between points subject to the constraint that you must travel along a certain surface, are *in general* called **geodesics**, and now we begin to see the connection.

When we wander around the Earth's surface like "bugs on a balloon," we imagine that [neglecting the odd bump here and there] we live in a 2-D space (North-South and East-West). In fact, we are simply restricted by practical considerations to a 2-D *surface* [within a few miles of altitude] "*embedded*" in a 3-D *space*. The analogous situation can arise for a 3-D **hypersurface** embedded in a 4-D *space-time* continuum. Such a hypersurface contains the **geodesics** along which light travels.

GR.4.3 Warp Factors

Before we go much further with the hard stuff, let's see if there is any way to *know* whether we are constrained to such a *curved* or "warped" [hyper]surface.

[3] You could open the door and look out, of course, but that would be cheating; besides, how do you know the view is not just an excellent illusion?

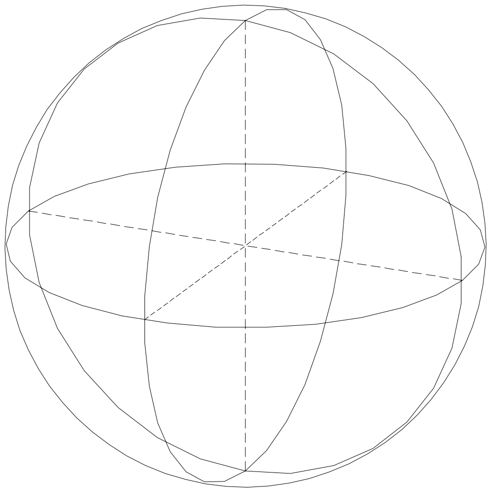


Figure 2 "Great circle" routes on the Earth are **geodesics** of the Earth's surface (a 2-D **hypersurface** *embedded* in 3-D **space**); geometrical figures drawn on this **hypersurface** do not obey Euclidian geometry!

For the "bug on the balloon" there certainly is: simply check whether Euclidean geometry (trigonometry, *etc.*) works properly on figures in the "plane" of the Earth's surface. As an extreme example, note that two "straight lines" which cross at one point on the Earth *will cross again* on the other side! Also note that one can make a "triangle" out of *great circles* in which *all three angles* are 90°! [Just make the length of each side equal to $\frac{1}{4}$ the circumference of the Earth.] And so on.

π as a Parameter

If we like, we can ever be quantitative about the degree of *curvature* of our embedded hypersurface. Picture the following construction: attach a string of length r to a fixed centre and tie a pencil to the other end. Keeping the string tight, draw a circle around the centre with radius r. Now take out a measuring device and run it around the perimeter to measure the circumference of the circle, ℓ. The ratio $\frac{\ell}{2r}$ can be defined to be Π. If the hypersurface to which we are confined is "flat," then Π will be equal to the value we know, $\pi = 3.14159\cdots$; but if we are on a *curved* (or "warped") hypersurface then we will get a "wrong" answer, $\Pi < \pi$.

Minkowski Space and Metrics

Star Trek notwithstanding, this is what is meant by "warped space." Our apparently "flat" (*i.e.* Euclidean) 3-D (x, y, z) universe is *embedded* in a 4-D (t, x, y, z) space called "**Minkowski space**." *Light* always follows a *geodesic* - the "shortest" distance between two points *constrained* to a given 3-D *hypersurface* - and *we can tell* if this hypersurface is *curved* in a 4-D analogy of the curvature of the Earth's 2-D surface in 3-D, because if it is, *Euclidean geometry will fail.*

This occurs (it turns out) *in any gravitational field.* Hence the terminology that has been popularized by various *SF* authors: "Gravity warps space."

Another way of putting this is to say that the **metric** of Minkowski space changes in a gravitational field.

A detailed mathematics of *tensor calculus* has been worked out to describe this effect quantitatively; I don't understand a bit of it, so you will be spared.

GR.4.4 Supernovae and Neutron Stars

Despite my ignorance, I can't resist trying to explain what happens in the presence of *really strong* gravitational fields. A typical scenario has a large sun (at least 10 times as big as ours, usually; relax!) cooling off until the gravitational attraction is strong enough to supply the *energy of confinement* necessary to overcome the **uncertainty principle** that normally prevents electrons from being confined inside protons. Then the reaction $e^-p \to \nu_e n$ (a sort of inverse neutron beta-decay) begins to convert hydrogen atoms to neutrons, emitting neutrinos as they go. The neutrons further enhance the gravitational energy density until there is a sudden chain reaction producing a **supernova** (the most violent explosion known) that blows off the exterior of the star (which is now rich in heavy elements because of all the neutrons being generated)[4] and leaves behind a **neutron star** - basically a giant atomic nucleus that doesn't fission because *gravity* holds it together.

Neutron stars are generally spinning very rapidly and have enormous magnetic fields "locked in" to their spin, so that the fields sweep up nearby charged particles and turn them into a beacon emitting electromagnetic radiation synchronized with the spinning star. Such beacons are "seen" on Earth as regularly pulsing radio sources or "**pulsars**," many of which are now known. Most nebulae (the remnants of supernovae) contain neutron stars at their cores.

The phenomenology of neutron stars is itself a huge and fascinating subject about which I know too little. Let's both go look them up and read more about them!

GR.4.5 Black Holes

If the neutron star is massive enough, then the gravitational force can grow strong enough even to overcome the hard-core repulsion between quarks and compress the neutrons themselves, making the gravitational force even stronger until *no force can resist* the **gravitational collapse**, at which point the entire mass of the star compresses (theoretically) to a single point called the **singularity**. We can't tell anything about the singularity for a simple reason: nothing that gets close to it can ever get away again.

The easy, handwaving way to see why is as follows: at any distance from a massive object, any other object will be *in orbit* about it providing it executes circular motion at just the right speed. As you get closer, the *orbital velocity* gets higher. Now, for a sufficiently heavy object, there is some radius at which the nominal orbital velocity is *the speed of light*. From inside that radius, called the **Schwarzschild radius** (r_S), *not even light can escape* but is inexorably drawn "down" into the singularity. Thus all light (or anything else!) falling on such an object's Schwarzschild radius will be perfectly absorbed, which accounts for the name, "**black hole**."

We can easily estimate r_S using a crude classical approximation: for a masss m in a circular orbit about a mass M, $F = ma$ gives $GMm/r^2 = mv^2/r$ which reduces to $GM/r = v^2$ or $r = GM/v^2$. If $v \to c$ this becomes $r_S = GM/c^2$. This result is actually off by a factor of 2: the actual Schwarzschild radius

[4] If it weren't for supernovae, there wouldn't be any heavy elements floating around the Galaxy to make planets out of and none of us would be here! Think of yourself as a sort of "supernova fossil."

is twice as large as predicted by this dumb derivation:

$$\text{True} \quad r_S = 2\frac{GM}{c^2}.$$

I have tried to find a simple explanation for this extra factor of two, but failed. Simply using the "effective mass" γm in place of m makes no difference, for instance, because it appears on both sides of the equation the same way. However, I don't feel too bad, because apparently it took Einstein about seven years to get it right. [The time it took him to develop his General Theory of Relativity, which explains that extra factor properly.]

A more rigourous description is beyond me, but I can repeat what I've heard and list some of the phenomenology attributed to **black holes**, of which there are two types: the **Schwatzschild** (non-rotating) black hole and the **Kerr** black hole, which *spins*. Presumably all real black holes are of the latter category, since virtually every star has some angular momentum, but there is probably a criterion for how *fast* it must spin to qualify as a Kerr black hole.

Schwarzschild Black Holes

One of the interesting features of **general relativity** is that *time slows down* as you approach the Schwarzschild radius of a black hole. Not to *you*, of course; your subjective experience of time is unaffected, but an outside observer would see your clock moving slower and slower (and turning redder and redder) as you fell into the black hole, until (paradoxically) you stopped completely (and were red-shifted out of sight) at r_S. Your own experience would depend upon the *mass* of the black hole. If it were big enough, the trip in free fall through r_S would be rather uneventful - you wouldn't notice much of anything unusual, unless of course you tried to get out again. If, on the other hand, you approached a *small* black hole, the *tidal forces* [the gravitational *gradient*] would tear you apart before you even reached r_S. This has some interesting consequences which I will discuss later.

The transformation between "outside" and "inside" coordinates has an interesting feature: while it is strictly impossible for anything *inside* r_S to come *out*, one can imagine extending the mathematics of the relativistic transformation from outside to inside, at least formally. The result would be that "inside time" is in the *opposite direction* from "outside time." This would mean that what we see as matter falling inexorably *into* a black hole must "look" to the interior inhabitants (if any) like an *expansion* of matter *away* from the singularity - a sort of **Big Bang**. Which raises an interesting question about *our* **Big Bang**: are we inside a **black hole** in someone else's Universe? Hmm.... And are the **black holes** in *our* Universe time-reversed **Big Bang**s for the inhabitants (if any) of their interiors? Hmmmmmm.... Unfortunately, this sophistry is probably all wrong. If you want a proper, correct and comprehensible description of phenomena at the Schwarzschild radius, go talk to Bill Unruh!

Kerr Black Holes

Well, moving right along, I should repeat what I've heard about **Kerr** (*spinning*) **black holes**. The problem with **Schwarzschild black holes** is, of course, that exploring them is strictly a one-way trip; once you pass through their Schwarzschild radius, you are doomed to fall right on in to the singularity.

Not so, apparently, with a **Kerr black hole** if it is spinning fast enough. Then the singularity is in a *ring* (sort of) and you can in principle plot a trajectory through the *middle* of the ring (or something like that) and *come out the other side*. Except that "the other side" may not have any resemblance to where-when you went in on this side! This has already been used as a great gimmick for *SF* stories involving time travel and other apparent logical paradoxes. I don't understand it at all, and I doubt very much that anyone else does, but one can always postulate that someone will, someday, and use it for practical(?) purposes. After all, as Arthur C. Clarke says, "Any sufficiently advanced technology is indistinguishable from magic."

Wormholes?

Another favourite gimmick of "hard *SF*" authors [those who try to make their stories consistent with the known "Laws of Physics"] is the **wormhole**, a sort of "space warp" analogous to the **black hole** but topologically more interesting. One can distort [*e.g.* fold] a 2-D surface (like a sheet of paper) embedded in a 3-D space until two apparently distant points are "actually" quite close together in the higher-dimensional continuum. Then a simple *puncture* across both sides will provide a "shrt cut" and drastically change the **connectedness** [a formal term in the mathematics of **topology**, believe it or not] of space. In a similar (?) fashion, one can imagine (?) a gravitational anomaly creating a "**wormhole**" making a "short cut" connection between two nominally distant regions of 3-D space. Great potential for space travel, right?

Sorry. John Archibald Wheeler, who has played a major rôle in the development of all this weird Gravitation stuff, proved a long time ago that *wormholes always pinch off* spontaneously before anything (even a signal propagating at the speed of light!) can get through them. Of course, this fact doesn't stop *Star Trek Deep Space 9* from having a lot of fun with the idea anyway.

Exploding Holes!

Another feature of *small* **black holes** is that they are *unstable*. This was explained in some detail by Bill Unruh in the UBC Physics Department. The basic idea is that for a *small* black hole the **tidal forces** at the Schwarzschild radius are so enormous that they can *tear apart the vacuum* - that is, pull one of the partners in a "virtual pair" or "bubble" down into the black hole while the other escapes as radiation. The resultant energy loss is deducted from the *mass* of the black hole, making it still *smaller*. This is a runaway process that ends in a rather impressive explosion. Not to worry, all the small "primordial" black holes (made in the **Big Bang**) have by now decayed. On the other hand, a *marginally larger* primordial black hole might have taken until now to get down to a size where the radiation really starts taking off. . . .

Mutability

What **conservation laws** do **black holes** respect? Not many. Mass-energy, angular momentum and electric charge are the only properties of what falls in that remain properties of the black hole itself. That means that all other "conserved" properties of matter, like **baryon number**, are "**mutable**" in the final analysis.

One consequence is that *protons* might experience **gravitational decay** in which they collapse into a *very* tiny black hole, only to immediately explode into (probably) a positron and some gamma rays. The estimated lifetime of protons against such a fate is $\sim 10^{45}$ years, which is not too worrisome.

Other consequences are more interesting, but only philosophically: the *interior* of a black hole [with which we can never communicate] may have entirely different properties - or even different "Laws of Physics" - than what we drop into it. Wheeler has taken this idea much further than I can follow, but it does make for interesting thinking. Good luck.

GR.4.6 Gravitational Redshifts and Twisted Time

In addition to the "ordinary" redshifts of distant stars caused by the relativistic Doppler shift due to the fact that they are actually receding from the observer on Earth, there is a **graviational redshift** of the light from *near* a large mass M when observed from a position *far* from the source, even if the source and observer are at rest relative to one another. This is not too surprising if we recall that a gravitational field has to be indistinguishable from an accelerated reference frame, and an accelerated object cannot be at rest for long! But an easier way to see the result is to remember that a massless particle like a **photon** still has an *effective mass* $m' = E/c^2$ where (if I may borrow a hitherto undemonstrated result from **quantum mechanics**) $E = h\nu$ for a photon. Here ν is the *frequency* of the light and $h = 6.626 \times 10^{-34}$ *J-s* is **Planck's constant**. Anyway, if the energy of a photon far from M is $E_\infty = h\nu_\infty$ (at $r \to \infty$) then its effective mass there is $m'_\infty = h\nu_\infty/c^2$ and as the photon "falls" toward M it should pick up kinetic energy until at a finite distance r its energy is $E = E_\infty + GMm'/r$ where the new effective mass is $m' = E/c^2$. Thus $E = E_\infty + (GM/c^2)E/r$ and if we collect the terms proportional to E we get $E_\infty = E(1 - r_\circ/r)$ where $r_\circ \equiv GM/c^2$. Dividing through by h/c^c gives the formula for the **gravitational redshift**,

$$\frac{\nu_\infty}{\nu} = 1 - \frac{r_S}{r} \qquad \text{where} \qquad r_S = 2\frac{GM}{c^2}.$$

(I have fudged in that extra factor of 2 that turns $r_\circ$ into the correct **Schwarzschild radius** r_S). This derivation is completely bogus, of course, but it does indicate why there is a gravitational redshift.

Given that any mechanism for generating electromagnetic waves constitutes a "clock" of sorts, the waves emitted by such a device constitute a signal from it telling distant observers about the passage of time at the origin. (Think of each wave crest as a "tick" of the clock.) The very existence of a **gravitational redshift** therefore implies that time passes slower for the clock that is closer to the mass - a result that was referred to earlier without proof.