THE UNIVERSITY OF BRITISH COLUMBIA

Physics 438 Assignment # 1: METABOLISM SOLUTIONS:

Tue. 09 Jan. 2007 — finish by Tue. 23 Jan.

The first part of this assignment was not for credit, but most people did it anyway! We appreciate those who Updated their Profiles, especially if they gave us their preferred Email address in the database. (To those who weren't able to get around to it in the first two weeks: it's never too late! The database awaits your input/revisions for the duration of the course.)

- **GET CONNECTED:** Go to the 438 Homepage at `http://musr.physics.ubc.ca/p438/` and
  - **Browse**. Make sure you know what's there, how to get to it and what it's good for (if anything). Email `jess@physics.ubc.ca` if you have any questions.
  - Make sure you can **log in** on that site and that there is *one and only one* entry for you in the 438 People Database. Your User ID and your initial password are both set to *your student number*. You may wish to change your password.[1] Your Profile in the 438 People Database has fields for all sorts of information about you, most of which are blank. You may want to Update your Profile, either to add/correct said information or to alter your privacy setting (at the bottom of the Update form).
  - Repeat the steps above for the 438 *WebCT* site[2] (to which there is a link on the 438 Homepage). This site is handy for certain things like Discussions, tentative Marks and your own personal 438 Homepage (if you'd like one); we may also use it for other things like chat or whiteboard work; if you have an idea for more creative uses of *WebCT*, please tell us about it.
  - **Send us an Email** containing a brief description of who you are, what talents and skills you bring to the 438 community, and what you hope to get out of the course. Please send this to `jess@physics.ubc.ca` *and* `jergold@zoology.ubc.ca` *and* `aweber@physics.ubc.ca` *from your own preferred Email address*, so we will all know how to contact each other in emergencies.

For the rest of the Assignment (and for all subsequent Assignments), *join a group of 3-5 students.* Please hand in one assignment per group and list the names & Email addresses of all group members at the top of each sheet.

In general, *if you think some necessary information is missing, make a reasonable assumption.* But always *write down* what that assumption is.

1. STAIRCASE OLYMPICS:

(*a*) Determine the mechanical power output[3] P_{walk} for each team member[4] walking up four flights of stairs (four floors) in the Hebb building. The height of one "floor" in the Hebb staircase is $h = 7.28$ m; if you do the exercise anywhere else you must measure h. By the definition of the mechanical efficiency $\eta \equiv P/\Gamma$, we have $P_{\text{walk}} = \eta_{\text{walk}} \times \Gamma_{\text{walk}}$ where $\Gamma_{\text{walk}} = b_{\text{walk}} \times \Gamma_0$ is the metabolic rate while walking up the stairs and b_{walk} is the corresponding metabolic activity factor. Calculate Γ_{walk} and b_{walk} for each team member. What value should you take for η? Discuss this choice and comment upon its validity in your written report.

ANSWER: Work is defined as the product of a force F times the distance x over which the force needs to be overcome. In our case the force to overcome is the gravitational force $F_g = M \times g$ over the height h. Now, the power P is the amount of energy used per time. In our case:

$$P_{\text{climbingstairs}} = \frac{M \times g \times h}{t} .$$

[1] Note that this ID/password combination is completely independent of any others you may already have memorized! Sorry about that, but ITservices still hasn't provided the third party CWL authentication they promised years ago.

[2] You must use your CWL (Campus-Wide Login) username/password to log in to any *WebCT* course.

[3] Note that we are focusing only on "useful work" (raising your mass against gravity); the work done by individual muscles moving body parts back and forth in walking along a *level* path is ignored here (*i.e.* it is included in Γ_{walk} but not in P_{walk}).

[4] Please do not attempt this exercise yourself if you are not sure you can perform the climb without excessive exertion or health risk. Each group should, however, have at least two climbing members.

This power needs to come from somewhere, which in the end is provided by the power produced by our metabolism. Since the metabolism needs also to produce power to keep your body functions working (*e.g.* heat), the total metabolic power produced during the climb is only partly used for the actual climb. Therefore

$$P_{\text{climbingstairs}} = \eta \times \Gamma_{\text{climbingstairs}}$$

where Γ is the metabolic rate and η the efficiency, which we don't know(!) but we have to assume some value, so we assume the "typical" value given in the textbook, namely 25%. Then the metabolic activity factor $b = \Gamma/\Gamma_0$ can be determined from Γ by assuming that Γ_0 is given by the "mouse to elephant" allometric relation.

For example, to calculate the metabolic activity factor b when $M = 60$ kg, $h = 7.3$ m and $t = 20$ s, we first find

$$\Gamma_{\text{climbingstairs}} = \frac{M \times g \times h}{t \times 0.25} = 858 \text{ W}$$

and then estimate the resting metabolic rate

$$\Gamma_0 = 4M^{0.75} = 86 \text{ W} .$$

Now, $\Gamma_{\text{climbingstairs}} = b \times \Gamma_0$, so the activity factor is $\boxed{b \approx 10}$.

(*b*) Estimate your uncertainty in this measurement.[5] The largest possible value, $P_{\max}$, is found by combining the largest likely value of your body mass, M_+,[6] and the shortest possible time Δt_-. Similarly combine the lowest likely body mass M_- and the longest likely time Δt_+ to find a value for $P_{\min}$. A good estimate of the uncertainty in your experimental result is thus $\delta P = (P_{\max} - P_{\min})/2$. Express your answer for the power P_{walk} in the form $P \pm \delta P$ W. (Always include *units*.) **ANSWER**: You have an uncertainty in your body mass (clothes, amount of water you have drunk, ...) which we assume for this example to be $\delta M = \pm 2$ kg and an uncertainty in time $\delta t = \pm 1$ s. We want to calculate the uncertainty in our calculation results. Using the prescription above with the example numbers, we get $\delta P = (P_{\max} - P_{\min})/2 = (233.7 - 197.8)/2 \approx 18$ W so that $\delta\Gamma_{\text{climbingstairs}} = \delta P/0.25$, giving $\boxed{\delta\Gamma_{\text{climbingstairs}} \approx 72 \text{ W}}$.[7]

(*c*) Determine $P \pm \delta P$ for each one of your team members when <u>running</u> up the stairs.[8] **ANSWER**: If the same person runs up the stairs in $t = 10$ s we get $\boxed{P_{\text{run}} = 429 \pm 57.2 \text{ W}}$ and $\boxed{\Gamma_{\text{run}} = 1716 \pm 228 \text{ W}}$, assuming the same uncertainties as before.

(*d*) Estimate the mass of the muscles M_{musc} used for running up the stairs and give the power to mass ratio $X_{\text{run}} = P_{\text{run}}/M_{\text{musc}}$ of these muscles.[9] Muscle mass (M_{musc}) = muscle volume (V_{musc}) times muscle density (ρ). Muscles have about the same density as water. **ANSWER**: The mean leg muscle mass of 60 kg women is measured to be 15 kg using X-ray absortiometry.[10] Thus the power-to-weight ratio for the running muscles (assuming the leg muscles do almost all the work) is $\boxed{X = P_{\text{run}}/M_{\text{musc}} = 29 \text{ W/kg}}$.

(*e*) Make a *table* including all your data showing `Name`, M, Δt_{walk}, P_{walk}, Γ_{walk}, Δt_{run}, P_{run}, Γ_{run}, Γ_0, V_{musc} and X_{run}, including the *uncertainty* in each. *Explain your most significant sources of uncertainty.* **ANSWER**: The tables will be different for each group. Uncertainties fall

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[5] This is often referred to as an "error estimate" but there is no negative connotation in estimating your uncertainty; it is not an "error" but merely an honest observation. Reporting measurements *without* any uncertainty is, by contrast, fundamentally *dis*honest!

[6] You probably know your "bare" weight from the Doctor's office, but did you have a heavy wallet in your pocket then?

[7] The general relation for a function $y(x_1, x_2, \cdots x_n)$ is

$$(\delta y)^2 = \sum_{i=1}^{n} \left(\frac{\partial y}{\partial x_i}\right)^2 \times (\delta x_i)^2 .$$

applied to a simple division, this gives

$$\left[\frac{\delta(a/b)}{(a/b)}\right]^2 = \left[\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2\right] .$$

For the example shown, this will give us an uncertainty $\boxed{\delta\Gamma_{\text{climbingstairs}} = 51.6 \text{ W}}$. Since this is just an *estimate*, for simple calculations you can get by with the method described in the question; for more complicated functions of many variables, that method will tend to <u>overestimate</u> the uncertainty, because not every "nudge" will be in the same direction. (This is why we add uncertainties "in quadrature".)

If you want to understand "error propagation" at a more sophisticated level, ask any of us to explain it.

[8] See the *caveat* for the first question.

[9] A good automobile engine generates about 1 kW/kg.

[10] Chilibeck *et al.*, Eur. J. Appl. Physiol. (1998).

into two categories: measurement imprecision and theoretical imprecision. In the former category are your elapsed time (probably measured to about ± 1 s) and your mass (probably estimated to about $\pm$1-2 kg, including clothes *etc.*). In the latter category is your efficiency η, which you had to *guess* in order to convert P into Γ, and your resting metabolism Γ_0, which you had to *assume* could be estimated from the "mouse-to-elephant" allometric relation, Γ_0[W] $= 4\ M$[kg]$^{0.75}$ — which we know is not a precise law but merely a consistent trend, with typical deviations of about a factor of two! A similar "slop" can be expected for the efficiency, which is only "typically" 25%. So you should not be expect these calculations to yield "true" values of your activity factor, were you to measure same more directly using oxygen consumption *etc.* As with all estimations of uncertainty, a good deal of *subjective judgement* is necessary; just be sure you can explain *why* you give the estimate you do.

Compound table for those who submitted data:

Weight	$\Delta t_{\rm run}$ [s]	$\Delta t_{\rm walk}$ [s]
51.8 kg	30	64
74.8 kg	18	60
79 kg	18	
57 kg		103
59 kg	32	65
140 lb	25	
135 lb	25	64
90 kg	69	110
135 lb	30	68
56 kg	24	106
120 lb	17	61

(*f*) Make a log-log graph for each of $P_{\rm walk}$, $P_{\rm run}$, $\Gamma_{\rm walk}$, $\Gamma_{\rm run}$, Γ_0 and $X_{\rm run}$ as functions of body mass M. **ANSWER**: This part will also be different for each group.

2. SO SWEET SO MEAN: A hummingbird weighing $M = 3.9$ g visits 1000 flowers daily and thereby collects nectar with an energy content of $\Delta H = 7 - 12$ kcal [see R. Conniff 2000][11]

(*a*) Take an average value of $\Delta H = 9 \pm 2$ kcal. What is the [sugar and][12] honey content of the nectar? (Honey has about 14/15 of the heat of combustion of sugar.)
ANSWER: Let's treat nectar as a solution of honey in water. The specific heat of combustion of glucose is $h_g = 4.15$ kcal/g, so the specific heat of combustion of honey is $h_h \approx h_g \times (14/15) = 3.87$ kcal/g. $\Delta H = \Delta M \times h_h$ so $\boxed{\Delta M \approx 2.3 \text{ g}}$.

(*b*) Determine the metabolic rate Γ of the little bird, and estimate its mechanical power output P. Assuming the metabolic rate function $\Gamma_0 = a\ M^{3/4}$ is applicable, determine the constant a in that rate function. **ANSWER**: Basal metabolic rate scales as $\Gamma_0 = a \times M^{0.75}$; the average daily metabolic rate for the hummingbird can be expressed as $\Gamma_{\rm daily} = \Delta H/\Delta t$ and therefore $\Gamma_{\rm daily} = 0.44$ W. using 4185 J/kcal with $\Gamma_{\rm daily} = a \times \Gamma_0$ and $M_{\rm hummingbird} = 0.0039$ kg, we find $a \approx 28$. Therefore $\boxed{P = \eta \times \Gamma_{\rm daily} = 0.11 \text{ W}}$, using again $\eta \approx 0.25$.

(*c*) Compare your calculated value of a with the constant $a_0 \approx 4$ of the mouse to elephant curve [Eq. (1.5) in the textbook] and determine the ratio $r = a/a_0$. Should this ratio be equal to the activity factor b calculated for the staircase run?
ANSWER: $\boxed{r \equiv a/a_0 \approx 28/4 = 7}$.
It is reasonable to assume that metabolic activity factors vary among species and also vary among different activities. However, $b \equiv \Gamma/\Gamma_0$ refers to the Γ_0 *for that animal*, so r is not b.

(*d*) What problems can you foresee for such a high metabolic rate? **ANSWER**: The high metabolic rate of hummingbirds is attributed to their characteristic hovering flight and small body size. This is probably accompanied by a very large daily energetic demand. Hummingbirds also posses the greatest mass specific power for muscle: 98 W/kg.[13]

(*e*) Calculate the specific metabolic rate $\gamma \equiv \Gamma/M$ for the hummingbird and for a 5-ton elephant. Which animal makes better use of the energy resources of the environment? **ANSWER**: The specific metabolic rate for the hummingbird is $\boxed{\gamma \equiv \Gamma_{\rm daily}/M = 113 \text{ W/kg}}$. For an elephant, assuming $M = 5000$ kg and

[11] R. Conniff, "So sweet, so mean" *Smithonian*, Sept. 2000, pp. 72-82.

[12] The wording of this question caused unnecessary confusion. We should have said either just "sugar content" or just "honey content". You get full credit for assuming either one (or any combination of the two, since the difference is slight). For this solution we will assume nectar is just honey diluted with water.

[13] Chai & Dudley, Nature (1995).

$\Gamma_0 = 4M^{0.75}$, $\gamma_0 = \Gamma_0/M = 0.48$ W/kg at rest, which even with an activity factor of $b = 10$ would not get higher then roughly $\boxed{5 \text{ W/kg}}$. Pound for pound, the elephant utilizes energy *far* more efficiently.

3. HOT DEFENCES: Giant hornets like to eat bee larvae and honey. They are so strong that they can just invade a beehive and kill the guards at the entrance and get at their favourite food. A certain strain of Japanese honey bees has found a thermodynamic defence. They can tolerate a temperature of 47.2°C (118°F). The hornets however can only stand 46.1°C (115°F). The bees have learned to raise their body temperature to 47.2° (117°F): they humm while contracting and relaxing their flight muscles, and only generate heat without producing external mechanical work ... and thereby steam the hornets in their own juice. Take a specific muscle power of $p = P/M = 150$ W/kg. The specific heat of tissue is close to that of water. Assume that the bees normally have a body temperature of 43°C and that 10% of the body weight of a bee is muscle.

(*a*) How much heat energy must be generated by each bee to reach the killing temperature? **ANSWER**: Heat energy: $\Delta Q = \Delta T \times c \times M_b$. The specific heat of tissue is about that of water: $c \approx 4.186$ kJ kg^{-1} K^{-1}. Thus with $\Delta T = 47° - 43°\text{C} = 4°\text{C}$ and $M_b \approx 0.0002$ kg, we have $\Delta Q = 4 \times 4.186 \times 10^3 \times 2 \times 10^{-4}$ or $\boxed{\Delta Q \approx 3.35 \text{ J}}$.

(*b*) What is the heating power of each bee? **ANSWER**: The mechanical power is the efficiency times the heating power: $P_{\text{mech}} = \eta \times \dot{Q}$. The specific muscle power $p = P_{\text{mech}}/M_m \approx 150$ W/kg, and so if $M_m \approx 0.1M_b$, with $M_b = 0.0002$ kg we get the bee's muscle weight $M_m \approx 2 \times 10^{-5}$ kg, giving a muscle power $P_{\text{mech}} = p \times M_m \approx 0.003$ W. Thus $\boxed{\dot{Q} = P_{\text{mech}}/\eta \approx 0.012 \text{ W}}$.

(*c*) How quickly do the bees reach the killing temperature? **ANSWER**: The flow of energy is identified with metabolic rate, in this case a heating metabolic rate: $\Delta Q = \Gamma_{\text{heating}} \times \Delta t$ with $\Delta Q = 4.8$ J. Again, $\Gamma_0 = 4 \times M^{0.75}$ and since $\Gamma_{\text{heating}} = b \times \Gamma_0$, $b = 15$ gives $\Gamma_{\text{heating}} = 0.1$ W. Thus $\Delta t = \Delta Q/\Gamma_{\text{heating}} \approx 48$ s or $\boxed{\Delta t = \Delta Q/\dot{Q} = 275 \text{ s}}$.

Alternatively: $\dot{Q} = c \times M_b(dT/dt)$; in this case, 0.012 W $=$ 4.186 J/kg $\times 0.0002$ kg $\times (dT/dt)$ giving $dT/dt = 0.0143°$C/s. Thus $0.0143°$C/s $\times \Delta t = 4°$C implies $\boxed{\Delta t = 279 \text{ s}}$.