THE UNIVERSITY OF BRITISH COLUMBIA

Physics 438 Assignment # 5: OPTICS SOLUTIONS:

Thu. 1 Mar. 2007 — finish by **Thu. 15 Mar.**

1. CATARACT OPERATION:

(*a*) BA's right eye was producing blurred images because the lens was getting "cloudy" and scattering some light. A cataract operation was performed where the natural lens was replaced by an artificial lens. . . .

The natural lens had $n = 1.413$ and radii of curvature $r_1 = 10$ mm and $r_2 = 7.8$ mm when focused at infinity. What was its focal length? **ANSWER**: There is some ambiguity to the question, inasmuch as the focal length of a lens depends upon the index of refraction of the medium in which it is immersed (see p. 273 of the textbook). However, when "the focal length" is discussed without reference to any medium, it is conventional to assume that the medium is vacuum or air with $n_o = n_i = 1$. In that case we can use the simple thin lens approximation

$$\frac{1}{f} = (n-1)\left[\frac{1}{r_1} + \frac{1}{r_2}\right] \qquad (1)$$

so that in this case the "optical power" is $1/f = 0.09425$ mm^{-1} and the focal length is $f = 10.61$ mm. But the LENSMAKER'S EQUATION says

$$\frac{1}{f} = (n-1)\left[\frac{1}{r_1} + \frac{1}{r_2} - \frac{(n-1)d}{nr_1r_2}\right] \qquad (2)$$

where d is the *thickness* of the lens along its central axis. Assuming the lens is about 5 mm in diameter, a quick sketch shows $d \approx 2$ mm, in which case the extra term in Eq. (2) reduces the optical power by about 0.003 mm^{-1} or a little over 3%; the true f is thus about 3% longer, or $\boxed{f = 10.96 \text{ mm}}$.

(*b*) It was replaced by an artificial symmetric lens with $r = r_1 = r_2$ made from crown glass ($n = 1.52$), what radius r gives the same focal length as before? **ANSWER**: First let's see what radius we get from Eq. (1): $1/10.96 = (1.52 - 1)[2/r] \Rightarrow$ $r = 10.96 \times 0.52 \times 2$ or $\boxed{r = 11.40 \text{ mm}}$. Another sketch suggests that this would make d even smaller. We could iterate the calculation with Eq. (2) to get a better approximation, but the result will certainly be within 2% of the above value.

(*c*) What should be the radius of curvature if the lens were made from silicate flint glass with $n = 1.65$? **ANSWER**: The same arithmetic with the new n gives $\boxed{r = 14.25 \text{ mm}}$.

2. IMAGING BY THE HUMAN EYE:

Find the distance i_1 of first image I_1 made by the human cornea ($r = 7.8$ mm) of an object placed at $o_1 = 250$ mm. Assume that the index of refraction of the material behind the cornea is $n = 1.336$. **ANSWER**: Here we have "only half a lens" so we set $r_2 = \infty$ and $d = 0$ to get $1/f_c = 0.0431$ mm^{-1} from Eq. (2). Now we use the all-purpose optics formula

$$\frac{1}{f} = \frac{n_o}{o} + \frac{n_i}{i} \qquad (3)$$

with $n_o = 1$ and $n_i = 1.336$ to get $1.336/i_1 = 1/f_c - 1/o_1 = 0.0431 - 1/250 = 0.0391$ or $\boxed{i_1 = 34.19 \text{ mm}}$.

This image serves as the (virtual) object (object distance $o_2 = -i_1$) from which the eye lens creates a real image I_2 on the retina at the image distance $i_2 = 20$ mm. The eye lens must contract to create a sharp image of such a close-distance object. Find the radius of curvature of the eye lens when focused at the object, assuming that the lens contracts symmetrically so that $r_{250} = r_1 = r_2$. **ANSWER**: We use Eq. (3) again to solve for the required focal length of the lens: $1/f = n_o/o_2 + n_i/i_2$ $= -1.336/34.19 + 1.336/20 = 0.0277$ mm^{-1} or $f = 36.07$ mm. We must now modify Eq. (1) to account for the index of the medium:

$$\frac{1}{f} = \left[\frac{(n_g - n_o)}{r_1} + \frac{(n_g - n_i)}{r_2}\right] \qquad (4)$$

with (in this case) $n_g = 1.413$, $n_o = n_i = 1.336$ and $r_1 = r_2 = r$, giving $0.0277 = 2(1.413 - 1.336)/r$ or $r = 2 \times 0.077/0.0277$ or $\boxed{r = 5.555 \text{ mm}}$.

3. FLY EYES:

Flies have compound eyes with many individual photo detectors. Assume the facet eyes of a certain fly consist of tiny light pipes of $d = 50$ μm diameter, length $L = 200$ μm and $n_e = 1.52$, mounted on a hemispherical shell of radius r_1 and capped by a conical structure of height $B = 50$ μm and $n_e = n_c = 1.52$. The top diameter of each cone is $D = 100$ μm. The light pipes and cones have cylindrical geometry. Typically $r_1 = 2$ mm. The tissue between these optical structures has $n_t = 1.33$.

(*a*) How many facet eyes are there in each compound eye if the cones touch each other?

ANSWER: The total area of the hemispherical surface is $A_t = 2\pi r_1^2 = 2.51 \times 10^{-5}$ m^2. The area of one cone top is $A_c = \pi(D/2)^2 = 0.785 \times 10^{-8}$ m^2. We can't just divide A_t by A_c because the facets are explicitly assumed to be circular, and a close-packed array of identical circles does not fill the entire area; there is a little "dead space" left over. If the array were square, the "filling factor" would be just $\pi/4$ (the ratio of a circle's area to that of a square with one side equal to the diameter of the circle); but the array is hexagonal, as can be seen from Fig. 8.24(b) on p. 293 of the textbook. In this case the filling factor is 90.69%, so the number of facets should be $N = 0.9069 A_t/A_c$ or $\boxed{N = 2902}$.

(*b*) What is the critical angle θ_{cr} of total internal reflection in the light pipe? **ANSWER**: In general $\sin\theta_{cr} = n_{out}/n_{in}$. In this case $n_{in} = n_e = 1.52$ and $n_{out} = n_t = 1.33$, giving $\sin\theta_{cr} = n_t/n_e = 0.875$ and $\theta_{cr} = 1.065$ radians or (in silly conventional units) $\boxed{\theta_{cr} = 61°}$.

(*c*) What is the acceptance angle β for the light pipe section by itself [see Fig. 8.23(b)]? **ANSWER**: Referring to Fig. 8.23 and following the instructions at the bottom of p. 291, we first convert $\gamma = 90° - \theta_{cr} = 29°$ into β using Snell's Law, $\sin\beta/\sin\gamma = n_e/1$ or $\sin\beta = 0.736$ giving $\boxed{\beta = 47°}$.

(*d*) What is the cone angle δ? **ANSWER**: From simple geometry, $\tan\delta = \frac{1}{2}(D-d)/L = 0.5 \times 50 \times 10^{-6}/200 \times 10^{-6} = 0.125$ giving $\boxed{\delta = 7.125°}$.

(*e*) Explain qualitatively why the acceptance angle β' for light pipe and cone [see Fig. 8.23(c)] is smaller than β. **ANSWER**: Without the *lens* focusing the light into the cone as in Fig. 8.24(a), a ray entering the front of the cone at an angle β will reach the inner surface of the cone at an angle of $\theta_{cr} - \delta$, which is too small for total internal reflection and will thus be lost. Only if the incoming angle is $\beta' = \beta - \delta$ or less will TIR be achieved in the cone section.

(*f*) Name 5 other animals that have compound eyes. **ANSWER**: Trilobites, spiders, crabs, shrimp and any other arthropods.

4. NOW YOU SEE ME, NOW YOU DON'T:

(*a*) Find and sketch 3 examples of optical illusions. Explain how the eye has been deceived in each case. **ANSWER**: There are thousands of examples; this was meant to be fun!

(*b*) Describe one example of an optical trick used by an animal to hide from or scare off its predators. **ANSWER**: Again, there are many examples: an animal may disguise itself as a leaf, stick or rock, if it is about the right size; it may look like another animal that is venomous or distasteful. The possibilities are endless.

(*c*) Describe an example of an animal that appears colorful due to either interference or diffraction, and explain in words what the "optical components" do to the light waves to generate the colors. **ANSWER**: Once again there are many examples, but of course our favourite is the blue (on one side) butterfly:

which achieves its brilliant colour by *thin film interference* in which the scales on its wings have a thickness just right for constructive interference between blue light reflected off both sides of the transparent scales and destructive interference between red light reflected off both sides.