THE UNIVERSITY OF BRITISH COLUMBIA

Science 1 Physics Assignment # 11:

THERMAL PHYSICS

Wed. 10 Jan. 2001 — finish by Wed. 17 Jan.

1. **"STAT - EC":** Consider the following simplified model of a sort of *stock market*: A given stock $\mathcal{S}$ has a total of N shares on the market for a fixed price ε. At a given time, n of these shares are bought and the remaining $N - n$ are unwanted. Thus the net investment in $\mathcal{S}$ is $U = n\varepsilon$. [Here ε and U are measured in *monetary* units, say dollars; I have used the same notation as for *energy* for reasons that will soon become evident.] To keep things simple, we shall assume that the *price* ε of a given stock *does not change*. Further, let's make the outrageous assumption that the stock market as a whole is *a priori equally likely to be found in any one of the fully specified states accessible to it* — *i.e.* that a given amount of capital is equally likely to be distributed amongst the various stocks in any of the possible ways that give the same total.[1]

(a) Invent a general definition for an economic analogue of *temperature* τ [measured in monetary units] that has the desired predictive power: that (given our starting assumptions) capital will tend to flow spontaneously from stocks with *higher* τ into others with *lower* τ and will stop flowing between two stocks only when they are in "economic equilibrium" — *i.e.* when they have the same "economic temperature" τ.

(b) Now assume that the entire market is in "economic equilibrium" and is so much larger than any of its parts that we may treat it as a "capital reservoir" $\mathcal{R}$ at an "economic temperature" of τ =\$100. Consider one share of one stock, valued at ε_1 =\$200: What is the probability that it will be bought at any given time?

(c) Assuming that $\mathcal{R}$ is also huge compared to the entire offering of $N_1 = 1000$ shares of stock $\mathcal{S}_1$ valued at ε_1 =\$200, what is the expected total investment U_1 in $\mathcal{S}_1$ when τ =\$100?

(d) If the economic temperature drops to τ =\$50, which stock will be likely to have the most capital U invested in it, $\mathcal{S}_1$ with $N_1 = 1000$ shares at ε_1 =\$200 per share or $\mathcal{S}_2$ with $N_2 = 1000$ shares at ε_2 =\$100 per share?

2. **ORTHO-** *vs.* **PARA-HYDROGEN:** Molecular hydrogen, H_2, consisting of two protons bound together with two electrons, can form in either the "singlet" state called *parahydrogen*, in which the total spin (intrinsic angular momentum) of the molecule is zero, or in any one of three "triplet" states of *orthohydrogen*, in which the proton spins combine to make a total spin of $1\hbar$ (the fundamental unit of angular momentum). For this problem, all you need to know is that the three triplet states are *degenerate* — *i.e.* they all have the same *energy* relative to the singlet state, namely $\varepsilon_3 = 2.375 \times 10^{-21}$ *J*. (The energy ε_1 of the singlet state can be taken to be zero, for reference.) Assume that the spin degrees of freedom of the H_2 molecules are unaffected by, but are in thermal equilibrium with, all their other degrees of freedom (like translational, rotational or vibrational). In this case, what *fraction* f_3 of H_2 molecules will be found (on average) in *ortho* states

(a) at room temperature (300 *K*)?

(b) at the boiling point of liquid nitrogen at atmospheric pressure (77 *K*)?

(c) at the freezing point of molecular hydrogen at atmospheric pressure (14 *K*)?

[1] This is not consistent with current economic theory, which focusses on "rational agents." Here we assume totally mindless, random investment decisions.