THE UNIVERSITY OF BRITISH COLUMBIA

Science 1 Physics Assignment # 13:

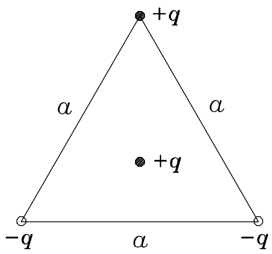
ELECTRIC CHARGE & GAUSS' LAW

Wed. 24 Jan. 2001 — finish by Wed. 31 Jan.

1. Units: Moving a charge of one 1 C [one *coulomb*] through a potential of 1 V [one *volt*] takes 1 J [one *joule*] of work (or will produce 1 J of kinetic energy). This handy relationship is a little clumsy in modern physics, which deals with elementary particles whose charge is quantized in units of $e = 1.602 \times 10^{-19}$ C. A convenient *energy* unit in accelerator physics is therefore the *electron volt* [eV], defined as the (kinetic) energy gained by an electron (or any other particle with the same size charge) when it is accelerated through a one volt [1 V] electric potential.[1] So ... **how many electron volts in one kilowatt-hour?**[2]

2. Earth's Extra Electrons: The Earth's atmosphere is a thin electrical insulator which has an excess of positive charge at higher altitude and an excess of negative charge near the Earth's surface, which acts like a spherical conductor. This charge separation gives rise to an approximately constant electric field pointing down through the atmosphere. However, *above the atmosphere* there is *no* electric field due to the Earth unless it has a *net* charge. Assuming that only electrostatic and gravitational forces are involved, *calculate how many excess electrons the Earth could hold.* (Hint: what will happen to a free electron just outside the atmosphere?)

3. Triangle of Charges: Three charges are placed at the vertices of an equilateral triangle of side a (as shown) and a fourth charge is placed at the centre of the triangle.



(a) *Sketch the electric field lines* inside and outside (nearby) the triangle.

(b) Derive an expression for the vector electric field $\vec{\boldsymbol{E}}$ at the *centre* charge due to the three other charges at the vertices of the triangle.

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[1] In case this is not self-evident, remember that the force $\vec{\boldsymbol{F}}$ on a charge q is given by $\vec{\boldsymbol{F}} = q\vec{\boldsymbol{E}}$, where $\vec{\boldsymbol{E}}$ is the electric *field*, measured in N/C or V/m. If we move a particle a distance $d\vec{\boldsymbol{x}}$ under the influence of a force $\vec{\boldsymbol{F}}$, that force does $\vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{x}}$ *work* on the particle — which appears as kinetic energy! If we move a charge $q = e$ a distance x *parallel* to a constant electric field E, the work done is eEx, or $(e)(E$ [V/m]$)(x$ [m]$)$ = eV. After all, the reason ϕ is *called* the "electric potential" is that when multiplied by q it gives the potential *energy* of the charge in the electric field.

[2] Recall that *watts* (W) are a unit of *power* (energy per unit time) equal to joules per second: 1 W $\equiv$ 1 J/s.

4. Gauss' Law for a Sheet of Charge: Imagine an infinite plane sheet of electric charge with $dQ/dA = \sigma$ units of charge per unit surface area. *Using your own words and drawings,*[3]

5. Gauss' Law for a Line of Charge: Imagine an infinite straight line of electric charge with $dQ/d\ell = \lambda$ units of charge per unit length. *Using your own words and drawings,*

(a) Using only (i) the SUPERPOSITION PRINCIPLE for the total electric field due to an assembly of electric charges, plus (ii) simple SYMMETRY arguments, deduce the *direction* of the electric field due to the line of charge.

(b) Using the preceding result plus the general ideas of GAUSS' LAW and simple geometry (surface area of a cylinder, *etc.*), deduce the dependence of the *magnitude* E of the electric field upon r, the *perpendicular distance* away from the line.

... *and* Tipler (4^{th} Edition)

Ch. **11**: problem **78**

Ch. **22**: problems **50**, **51**, **54** and **82**

Ch. **23**: problems **10**, **53** and **57**

WOOPS![4]

Ch. **18**: problems **33**, **44** and **62**

Ch. **19**: problems **45**, **66** and **96**

Ch. **20**: problem **19**

[3]For this and the next question, the final result is well known and the full derivation can be found in any textbook; however, you will not really *own* GAUSS' LAW until you have expressed and used it *your own way*. This is true for almost everything, but GAUSS' LAW is so important (and so *simple!*) that you should not miss this chance to make it yours.

[4]We forgot to hand out last week's assignment on Thermodynamics! These were the problems; see if you can find time to do them over then next few weeks — no deadline, but it would be wise to have a look before the Physics Midterm.