THE UNIVERSITY OF BRITISH COLUMBIA

Science 1 Physics Assignment # 18:

Maxwellativity

Wed. 21 Mar. 2001 — finish by Wed. 28 Mar.

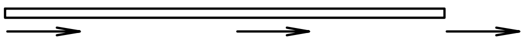
1. **SIMULTANEITY:** At $t = t' = 0$, $x = x' = 0$, events A and B occur in reference frame O at $t_A = 0.3\mu$s, $x_A = 150$ m and $t_B = 0.4\mu$s, $x_B = 210$ m, respectively. *Where* (x'_A) *and when* (t'_A) do these events occur in reference frame O' if

(a) O' moves at a velocity of $0.6c$ in the positive x direction relative to O?

(b) O moves at a velocity of $-0.6c$ in the positive x direction relative to O'?

(c) Could these two events be *simultaneous* in some other reference frame? If so, what must be the velocity of that frame relative to O?

2. **RELATIVISTIC RUBBER BAND:**



A certain piece of elastic can be stretched to twice its unstretched length before it breaks. At time $t = 0$ it is at rest in the unstretched state, laid out straight along the x axis. Thereafter all parts of it are accelerated longitudinally (in the x direction) with a constant acceleration of one "gee" ($a = 9.81$ m/s^2) as measured in the lab frame (the frame where it was originally at rest). At what time t does the elastic break?

3. **VELOCITY TRANSFORMATION:** An observer on Earth observes two spacecraft moving in the *same* direction toward the Earth. Spacecraft A has a speed of $0.5c$ and spacecraft B has a speed of $0.8c$ in the Earth's frame. What is the speed of spacecraft A as measured by an observer in spacecraft B?

4. **TWIN "PARADOX":** Frank and Mary are twins. Mary jumps on a spaceship and goes to a hypothetical star 8 lightyears [ly] away and returns.She travels at a speed of $u = 0.8c$ with respect to the Earth in both directions and spends a negligible fraction of her time accelerating and decelerating.Mary sends out a radio signal to Earth every year (her time). Frank also sends out a radio signal every year (his time) to the spaceship.

(a) How many signals does Mary receive from Frank before she turns around at the other star?

(b) At what time (Earth time) does the frequency at which Frank receives signals suddenly change? How many signals has he received by this time?

(c) What is the total number of signals each twin receives from the other?

(d) How much time does the trip take, according to each twin?

(e) How much time does each twin claim the *other* twin will have measured for the trip? Who is right?

Hint: see diagram and table on reverse.

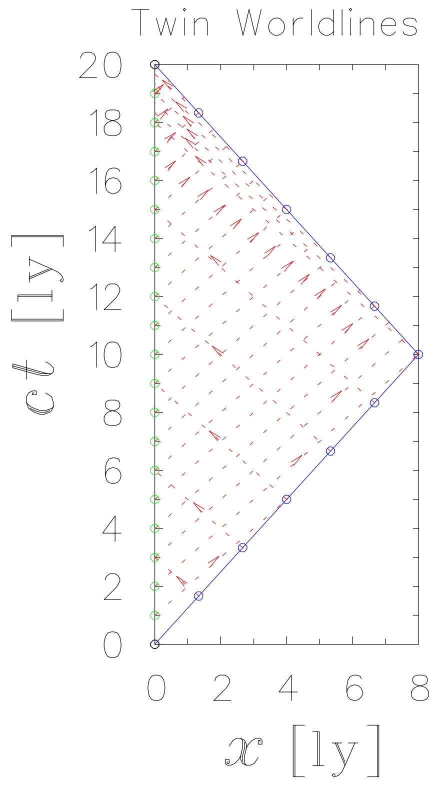


Figure for Twin "Paradox":
The trajectories (worldlines) of Frank, Mary and their light signals are shown in Frank's reference frame. (Frank's worldline is just a vertical line at $x = 0$.) The slopes of Mary's worldlines are $\pm\beta^{-1} = \pm c/v$. The worldlines of all light paths (dotted lines) have slope ± 1. As viewed by Frank, Mary's clock is slow by a factor γ throughout the trip; thus the times at which she sends signals are more than one year apart in Frank's reference frame. Note that one always has to choose one reference frame in which to draw all events!

T&R TABLE 2.1 *Twin Paradox Analysis*

after A. French, *Special Relativity*, p. 158 (W.W. Norton, New York, 1968)

Item	**measured by Frank (remains on Earth)**	**measured by Mary (traveling astronaut)**
Time of total trip	$T = 2L/v$	$T' = 2L/\gamma v$
Total number of signals sent	$\nu T = 2\nu L/v$	$\nu T' = 2\nu L/\gamma v$
Frequency of signals received at beginning of trip ν'	$\nu\sqrt{\dfrac{1-\beta}{1+\beta}}$	$\nu\sqrt{\dfrac{1-\beta}{1+\beta}}$
Time of detecting Mary's turnaround	$t_1 = L/v + L/c$	$t_1' = L/\gamma v$
Number of signals received at the rate ν'	$\nu' t_1 = \dfrac{\nu L}{v}\sqrt{1-\beta^2}$	$\nu' t_1' = \dfrac{\nu L}{v}(1-\beta)$
Time for remainder of trip	$t_2 = L/v - L/c$	$t_2' = L/\gamma v$
Frequency of signals received at end of trip ν''	$\nu'' t_2 = \dfrac{\nu L}{v}\sqrt{1-\beta^2}$	$\nu'' t_2' = \dfrac{\nu L}{v}(1+\beta)$
Total number of signals received	$2\nu L/\gamma v$	$2\nu L/v$
Conclusion as to other twin's measure of time taken	$T' = 2L/\gamma v$	$T = 2L/v$