Some Simple Derivatives

- **Definition:** The rate of change [slope] of a function at a point is the limiting value of its *average* slope over an interval including that point, as the width of the interval shrinks to zero:

$$\frac{dy}{dx} \equiv \lim_{\Delta x \to 0} \frac{y(x + \Delta x) - y(x)}{\Delta x}$$

All the remaining Laws and Rules can be proven by algebraic manipulation of this definition.

- **Power Law:**

$$\frac{d}{dx}(x^p) = p\, x^{p-1}$$

valid for *all* powers p, whether positive, negative, integer, rational, irrational, real, imaginary or complex.

- **Product Law:** The derivative of the product of two functions is *not* the product of their derivatives! Instead,

$$\frac{d}{dx}\left[f(x) \cdot g(x)\right] = \frac{df}{dx} \cdot g(x) + f(x) \cdot \frac{dg}{dx}$$

- **Constant times a Function:** The *Product Law* gives

$$\frac{d}{dx}\left[a\, y(x)\right] = a\, \frac{dy}{dx}$$

where a is a constant (*i.e.*, not a function of x). This is often referred to as "pulling the constant factor outside the derivative."

- **Function of a Function:** Suppose y is a function of x and x is in turn a function of t.

$$\frac{d}{dt}\, y[x(t)] = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

(also known as the **Chain Rule**)

- **Exponentials:**

$$\frac{d}{dx}\left[e^{k\,x}\right] = k\, e^{k\,x}$$

where k is any constant.

- **Natural Logarithms:**

$$\frac{d}{dx}\left[\ln x\right] = \frac{1}{x}$$