the University of British Columbia

# Science 1 Physics

# Sessional Examination

20 April 1999

Time: $2\frac{1}{2}$ hours

FULL NAME: ______________________ STUDENT # : ______________

SIGNATURE: ______________________

This Examination paper consists of 13 pages (including this one). Make sure you have all 13.

instructions:

Write your name on every sheet.

No memory aids allowed.

marking:

| | | | |
|---|---|---|---|
| **Q1** | /30 | **Q5** | /12 |
| **Q2** | /12 | **Q6** | /10 |
| **Q3** | /12 | **Q7** | /12 |
| **Q4** | /12 | **TOT** | /100 |

Names of Instructors: Jess H. Brewer & Domingo Louis-Martinez

**Q1 "QUICKIES"** [30 marks — 5 each]

(*a*) If $\vec{\boldsymbol{g}}$ is the acceleration of a small test mass due to the gravitational attraction of a large accumulation of other mass, we can write GAUSS' LAW FOR GRAVITY in the form $\oint\!\!\oint \vec{\boldsymbol{g}} \cdot d\vec{\boldsymbol{A}} = k_G M_{\text{encl}}$, where $M_{\text{encl}}$ is the total mass inside the closed surface over which $\vec{\boldsymbol{g}} \cdot d\vec{\boldsymbol{A}}$ is integrated. *What is the value of the constant* $k_G$?

(*b*) A velocity selector has a magnetic field $\vec{\boldsymbol{B}}$ perpendicular to an electric field of 20,000 V/m. We find that charged particles with velocity $v = 4.0 \times 10^4$ m/s perpendicular to both $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$ pass through the device undeflected. What is the strength of $\vec{\boldsymbol{B}}$ [in Tesla]?

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(*c*) If $\epsilon_\circ$ and $\mu_\circ$ are respectively the permittivity and the permeability of free space, what is the value [including *units*] of the quantity $(\mu_\circ \epsilon_\circ)^{-1/2}$ and *what does it represent*?

(*d*) An electric dipole, consisting of a 0.01 C positive electric charge 1 cm away from an equal-magnitude negative charge, is located at the centre of a sphere of radius 1 m. What is the average value of the dipole's electric field normal to the sphere's surface? Explain.

*(e)* An oscillator consisting of a coil, a capacitor and a 1000 $\Omega$ resistor, all connected in series, has a resonant frequency of $f_\circ = (1/2\pi)\ kHz$ and a decay time of $\tau = 0.1\ s$. What is the inductance $L$ of the coil?

*(f)* What is the *total relativistic energy* of

*i)* an *electron* with a kinetic energy $K_e = 0.511$ MeV? $E_e =$ ______________ MeV.

*ii)* a *photon* with a momentum of $p_\gamma = 0.511$ MeV/c? $E_\gamma =$ ______________ MeV.

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## Q2 Maxwell's Equations [12 marks]

Match up the names on the left and the principles on the right with the equations in the middle. (For each match, draw a connecting line.) **Circle** all that refer to **Maxwell's equations**.

| | | |
|---|---|---|
| Lorentz Force | $\oiint_S \vec{\boldsymbol{D}} \cdot d\vec{\boldsymbol{A}} = Q_{\rm encl}$ | A charged particle follows a curved path in a magnetic field. |
| Gauss' Law for Magnetostatics | $\vec{\boldsymbol{F}} = q\left(\vec{\boldsymbol{E}} + \vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}\right)$ | "Lines of force" are continuous (unbroken) except where they begin and end on *charges*. |
| Gauss' Law for Electrostatics | $\oint_C \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{\ell}} = -\frac{\partial}{\partial t}\iint_S \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{S}}$ | The "circulation" of magnetic field "lines" around an *open* surface is proportional to the net electric current flowing through that surface. |
| Ampère's Law | $\oiint_S \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}} = 0$ | There are (apparently) no "magnetic monopoles." |
| Faraday's Law | $\oint_C \vec{\boldsymbol{H}} \cdot d\vec{\boldsymbol{\ell}} = I_{\rm encl} + \frac{\partial}{\partial t}\iint_S \vec{\boldsymbol{D}} \cdot d\vec{\boldsymbol{S}}$ | Changing an electric field generates a magnetic field. |
| Displacement Current | | Changing a magnetic field generates an electric field. |
| | | Moving charges generate magnetic fields. |

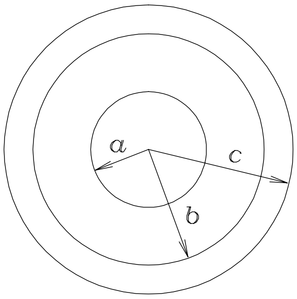
Definitions: $\vec{\boldsymbol{D}} \equiv \epsilon\vec{\boldsymbol{E}}$ $\quad \epsilon \equiv \kappa\epsilon_\circ$ $\quad \kappa =$ dielectric constant.

$\vec{\boldsymbol{B}} \equiv \mu\vec{\boldsymbol{H}}$ $\quad \mu = \mu_\circ(1+\chi_m)$ $\quad \chi_m =$ magnetic susceptibility.

(In vacuum, $\kappa = 1$ and $\chi_m = 0$.)

YOUR FULL NAME:

## Q3 Concentric Spheres [12 marks]




A solid insulating sphere of radius $a = 10$ cm is uniformly charged and placed at the centre of a conducting spherical shell of inner radius $b = 20$ cm and outer radius $c = 25$ cm, as shown. The electric field at a point 15 cm from the common centre is measured to be 1600 N/C directed radially inward, while the electric field at a point 50 cm from the centre is 200 V/m radially outward. From this information, find

($a$) [4 marks] the net charge on the insulating sphere;

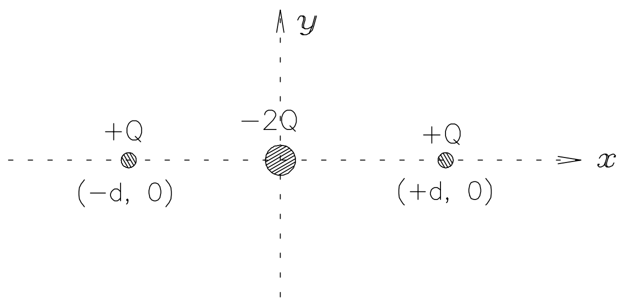
($b$) [4 marks] the net charge on the hollow conducting sphere;

($c$) [4 marks] the total charge on *each* of the inner and outer *surfaces* of the hollow conducting sphere.

YOUR FULL NAME:

**Q4 Linear Quadrupole** [12 marks]

The arrangement of charges shown is referred to as a *linear quadrupole*.

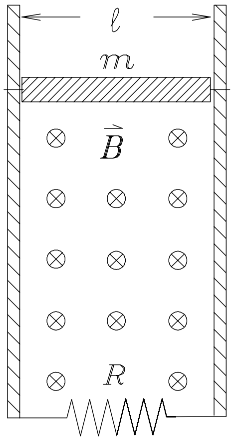



(*a*) [4 marks] Draw *either* the equipotentials *or* the electric field lines around the quadrupole.
I have drawn ☐ equipotentials ☐ electric field lines.

(*b*) [4 marks] Find the total electrostatic energy of this system of charges.

(*c*) [4 marks] Show that the electrostatic *potential* at a point $(x, 0)$ on the $x$ axis drops off approximately as $x^{-3}$ for $x \gg d$.

## Q5 Falling Bar [12 marks]




A horizontal bar of mass $m$ is free to slide without friction down the vertical rails of a conducting frame, as shown. The combined resistance of the bar and the frame is negligible compared to $R$, the resistance placed in series with this circuit. What is the *terminal speed* of the bar as it falls under the influence of gravity (at the surface of the Earth) through a uniform horizontal magnetic field $\vec{\boldsymbol{B}}$ directed perpendicular to the plane of the frame? (Give your answer in terms of $m$, $B$, $\ell$, $R$, $g_\circ$ and other fundamental constants.)

## Q6 Relativistic Rocket [10 marks]

A rocket moves at constant speed $u$ toward a stationary mirror. When the rocket is a distance $d$ from the mirror (as measured by an observer at rest with respect to the mirror) a light pulse is emitted from the rocket toward the mirror and is subsequently reflected back to the rocket. What is the *total travel time of the light pulse* as measured by an observer in

$(a)$ [5 marks] the rest frame of the mirror?

$(b)$ [5 marks] the rest frame of the rocket?

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**Q7 YOUR CHOICE:** [12 marks]

Answer **ONE** of the following **THREE QUESTIONS**:

### Q7.1 Lorentz Velocity Transformations

Object $A$ is moving in the $x$ direction at velocity $v_A$ in the rest frame of observer $S$. Observer $S'$ is moving in the $x$ direction at velocity $u$ relative to observer $S$. **Derive** a formula for the velocity $v'_A$ of object $A$ in the rest frame of observer $S'$.

(*Hint: Differentiate* the Lorentz transformations for time $t$ and position $x$.)

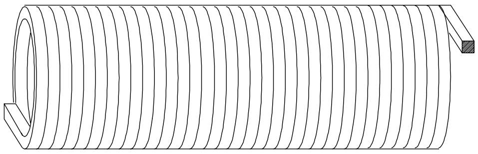
**Q7.2 Cyclotron Design** — Answer EITHER this question OR the one on the previous page OR the one on the next page!

(*a*) [6 marks] Show that the orbital period $T$ of a charged particle moving perpendicular to a uniform magnetic field $B$ is independent of the particle's velocity $v$ as long as $v \ll c$ (where $c$ is the speed of light).

(*b*) [6 marks] Show that when a particle of rest mass $m$ is accelerated to kinetic energy $K$, its orbital period is multiplied by a factor $\left(1 + \dfrac{K}{mc^2}\right)$.

YOUR FULL NAME:

**Q7.3 Solenoid Design** — Answer EITHER this question OR one of the two preceding!



You are given a certain *volume* $V$ of copper from which to make *wire* with a square cross section. The wire you make is then used to wind a *one-layer solenoid* as shown. (Ignore the practical necessity for insulation between the wires and pretend that they can be wound so that the distance between turns is exactly the width of the wire.) *Your goal is to generate the highest possible magnetic field at the centre of the solenoid.*

($a$) [6 marks] For wire of a given cross section and for a given *current* $I$ in the wire, should you make the *inside diameter* of the solenoid large or small? Explain.

($b$) [6 marks] For a given inside diameter and for a given *voltage* $\mathcal{E}$ applied to the solenoid, should you make the wire *long and thin* or *short and thick*? Explain.

Table 1 — **Constants and Conversion Factors.** (You may not need all of these!)

| | | | |
|---|---|---|---|
| *Acceleration due to gravity* | $g_\circ$ | $=$ | $9.81$ m-s$^{-2}$ (at Earth's surface) |
| *Mean radius of Earth* | $R_E$ | $=$ | 6367 km |
| *Permittivity of free space* | $\epsilon_\circ$ | $=$ | $8.854 \times 10^{-12}$ F/m [1 F $\equiv$ 1 C/V $\equiv$ 1 C$^2$/N-m] |
| *constant in Coulomb's Law* | $k_E = \dfrac{1}{4\pi\epsilon_\circ}$ | $=$ | $8.988 \times 10^9$ N-m$^2$/C$^2$ |
| *Permeability of free space* | $\mu_\circ$ | $=$ | $1.2566 \times 10^{-6}$ H/m $=$ $4\pi \times 10^{-7}$ N/A$^2$ |
| *constant in Biot-Savart Law* | $k_M = \dfrac{\mu_\circ}{4\pi}$ | $=$ | $10^{-7}$ T-m/A |
| *Electric charge of a proton* | $e$ | $=$ | $1.602 \times 10^{-19}$ C |
| *Speed of light in vacuum* | $c$ | $=$ | $2.997 \times 10^8$ m/s |
| *Proton rest mass* | $M_p$ | $=$ | $1.67 \times 10^{-27}$ kg $=$ 938 MeV/$c^2$ |
| *Electron rest mass* | $m_e$ | $=$ | $9.11 \times 10^{-31}$ kg $=$ 0.511 MeV/$c^2$ |
| *electron volt* | 1 eV $= 1.602 \times 10^{-19}$ J $= 1.602 \times 10^{-12}$ erg $= k_{\rm B} \times 11,600$ K | | |

## Mathematical Formulae

$$(1+\delta)^{-1} = 1 - \delta + \delta^2 - \delta^3 + \delta^4 - \cdots$$

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