Science 1 Physics
Christmas Exam - 2000

- 2.5 hours -

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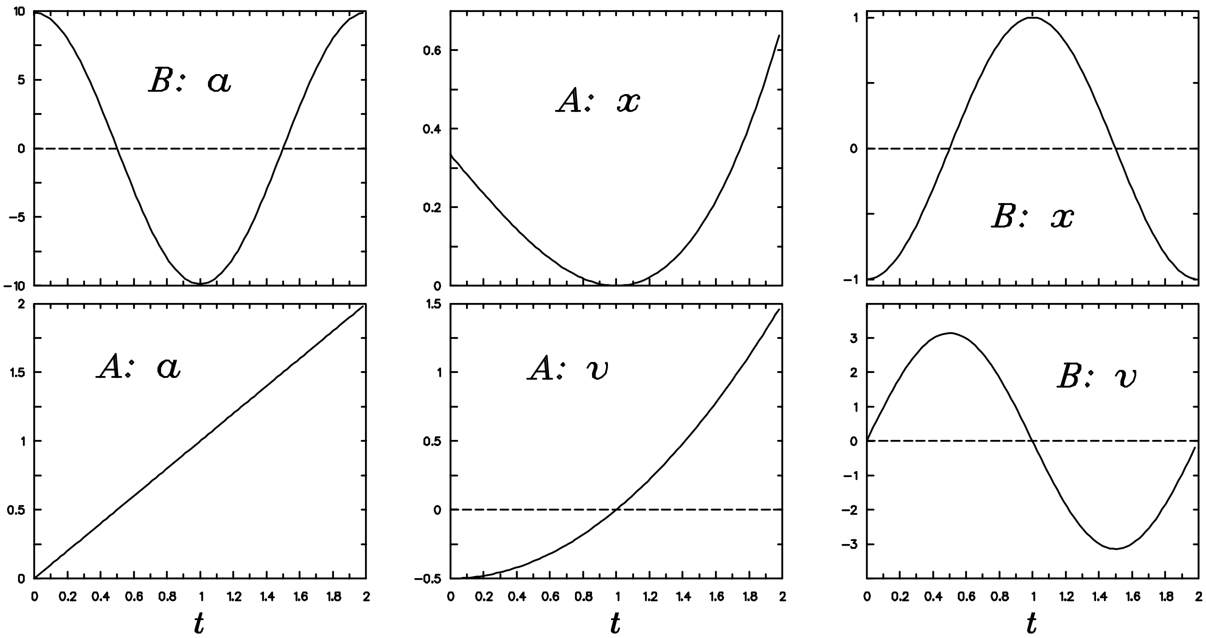
1. **"QUICKIES"** [30 marks — 6 each]

   (a) Shown below are six graphs describing the motion of two different objects, A and B, in one dimension. The horizontal axis is the same for each graph and represents *time* over the range from 0 to 2 s. The vertical axes are either distance x [in m], speed v [in m/s] or acceleration a [in m/s^2], with only the numerical values indicated. There is one and only one graph of each of $x(t)$, $v(t)$ and $a(t)$ for each of the two objects.



   i. *Which is which*? Indicate on each graph which function it describes for which object [*e.g.* "A: $v(t)$"]. One graph is already labelled with an A to get you started. **ANSWER:** See graph.

   ii. Describe the motion of both A and B <u>in words</u>. (No numbers or algebraic symbols!) **ANSWER:** The acceleration of A increases linearly with time. (This is known as "constant jerk".) Its initial acceleration is zero, its initial velocity is negative and its initial position is positive; thus it initially moves back toward the origin, which it reaches after one second, and then turns around and accelerates away again.
   The motion of B should be immediately recognized as SHM; it remains only to define the period (two seconds), the amplitude (one meter) and the initial phase (pi). Remember, each time you take a derivative (*e.g.* to get the velocity or the

acceleration) you multiply by the frequency, which is larger than unity, so the smallest vertical excursions must be for the displacement itself.

(b) What is the orbital radius of a geosynchronous satellite (one which is always directly over the same point on the Earth's equator)?

ANSWER: The FBD for the satellite is trivial: there is a single force $F = \frac{GMm}{r^2}$ acting to cause a centripetal acceleration of $a = r\omega^2$; Newton's Second Law thus demands $mr\omega^2 = \frac{GMm}{r^2}$ or $r^3 = \frac{GM}{\omega^2}$. Since $\omega = \frac{2\pi}{1 \text{ day}} = 7.272 \times 10^{-5} \text{ s}^{-1}$, plugging in G and M from the constants provided gives $\boxed{r = 4.224 \times 10^7 \text{ m or } 42{,}241 \text{ km.}}$

For some reason lots of people felt the Earth's radius should be subtracted from this value.

Quite a few people used one *year* as the orbital period. ?!

Others calculated the velocity and centripetal acceleration of a point on the Earth's surface for some reason.

(c) Suppose that a 100 W source radiates light of wavelength 600 nm uniformly in all directions and that the eye can detect this light if at least 20 photons per second enter a dark-adapted pupil 6 mm in diameter. *How far from the source* can the light be detected by eye?

ANSWER: This question required that you remember one of the formulae for the energy of a photon, either $E = hf$ (where f is the frequency in Hertz) or $E = hc/\lambda$ where λ is the wavelength of the light, c is the speed of light and h is Planck's constant (provided on the last page). This was the easy part, giving $E = 3.31 \times 10^{-19}$ J and implying that the source is emitting $N = 3.02 \times 10^{20}$ photons per second. The harder part was then realizing that these N photons are spread all over the surface of a sphere at radius R, a total area of $4\pi R^2$, of which only πr^2 (where $r = 0.003$ m is the radius of the pupil) is intercepted by the eye's pupil. Thus we want $\frac{20}{N} = \frac{\pi r^2}{4\pi R^2}$, which you can then solve for $\boxed{R = 5.83 \times 10^7 \text{ m} = 5830 \text{ km.}}$

This was meant to be a conceptually challenging problem, but not a calculationally complex one. Hence its place in the "Quickies" section.

(d) The positive muon (μ^+) is an unstable elementary particle with a constant decay rate of $\lambda = 1/\tau_\mu$, where $\tau_\mu = 2.197\ \mu$s is the mean muon lifetime. The M15 muon beam at TRIUMF deposits an average of 10^6 positive muons in a target every second.

i. What is the average number of these muons present in the target at any time?

ANSWER: Again this is a simple problem once you get the idea, but many didn't. In steady state (after the beam has been going for a long time compared to microseconds) the number N_0 of muons present in the target is (on average) constant. Thus $dN/dt = 0$ when $N = N_0$. But a simple Chemistry-style rate equation says $dN/dt = I - \lambda N$ where $I = 10^6 \text{ s}^{-1}$ is the rate at which muons are supplied by the beam. So in steady state we must have $\lambda N_0 = I$ or $\boxed{N_0 = I\tau_\mu = 2.197 \text{ muons.}}$

For some reason a few people thought it didn't make sense to have a non-integer number of muons on average, so they rounded down to 2. ??

ii. If the beam is shut off suddenly at time t_0, at what time is it most probable that there will be *exactly one* muon in the sample?

ANSWER: This part is just plain exponential decay: $N(t) = N_0 e^{-t/\tau_\mu}$ with $N_0 = 2.197$ and the given value of $\tau_m u$. It's simplest to leave times in microseconds for this part. Thus to get $N = 1$ we set $1 = 2.197 e^{-t/2.197}$ and take the log of both sides, giving $\boxed{t = 1.729\ \mu\text{s.}}$

(e) Which of the following features are *essential* for SIMPLE HARMONIC MOTION?

$\underline{\text{A linear restoring force}}$.	$\underline{\text{Newton's First Law}}$.	$\dot{x} = -kx$.
Viscous damping.	$\underline{\text{A quadratic potential minimum}}$.	A spring.
$\underline{\ddot{x} = -\omega^2 x}$.	A driving force at the resonant frequency.	

2. **Stick-Ball Sweet Spot** [20 marks] Any sort of rigid "bat" has a "sweet spot" where you want to hit the ball, because if the ball hits the bat at that point you will not feel any impact in your hands where you hold the bat.

(a) Explain why this is so. Make your explanation simple and qualitative but clear and complete enough to convince a skeptic. A diagram may be very helpful, but numbers are irrelevant.

ANSWER: What matters here is whether there is any sudden *acceleration* of the end of the bat where you hold it. Call this point A and call the centre of mass C. The position of the "sweet spot" S is a distance x from A, whereas C is a distance $L/2$ from A for a uniform rod. The force F exerted by the ball on impact at S causes an acceleration of C given by $a_c = F/M$ where M is the mass of the bat. At the same time, F applied at S produces a torque $\tau_c = -(x - L/2)F$ about the centre of mass. This torque causes an angular acceleration $\alpha = \tau_c/I_c$ where $I_c = \frac{1}{12}ML^2$ is the bat's moment of inertia about its centre of mass. The net acceleration of point A is the sum of the acceleration of C and the tangential acceleration of A relative to C due to α. When these two exactly cancel, $a_A = 0$ and there is no "jerk" at A.

(b) For an idealized "stick bat" (a thin, uniform rigid rod) of length L, calculate the *position* of the "sweet spot" (assuming you can hold it by the very end).

ANSWER: Putting together the above equations gives $a_C = 0 = \dfrac{F}{M} - \dfrac{(x - L/2)F}{\frac{1}{12}ML^2}$ or $1 = \dfrac{12(x - L/2)}{2L}$ or $\frac{1}{6}L = x - L/2$ or $x = (\frac{1}{2} + \frac{1}{6})L$ or $\boxed{x = \dfrac{2}{3}L.}$
Several people guessed this answer. I gave some credit for good physical intuition.

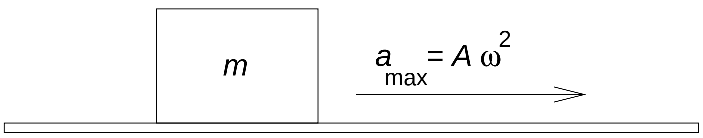
This question was meant to be conceptually challenging, but it was surprising that only one person got it entirely right. Very few even saw the basic principle involved, demonstrating that an ability to reproduce a variety of practiced problem solutions is not the same as a real grasp of the principles of rigid body dynamics, which is necessary to solve even a simple problem of an unfamiliar type. This should not humiliate anyone, but rather give everyone a deeper respect for the subtlety of "simple Mechanics".

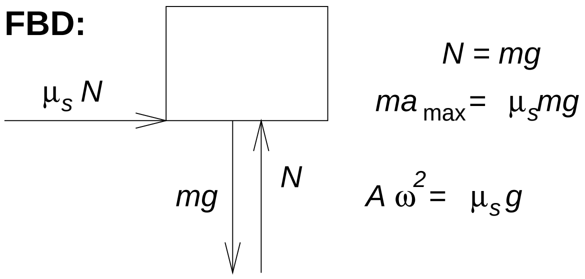
Another surprise was the number of people who evidently have never played any form of ball-and-bat game, and so had no physical intuition at all about the problem.

There were, as always, many who began hunting blindly for the right formula before asking, "What's this really all about?" You *cannot* do Physics this way. I hope this is now clear.

3. **Don't Slip!** [15 marks] A level platform vibrates horizontally in simple harmonic motion with a period of 0.8 s. A box on the platform starts to slide when the amplitude of the vibration reaches 40 cm. What is μ_s, the coefficient of static friction between the box and the platform?

   ANSWER: See sketch and FBD below. The box first slips when the maximum acceleration of the platform is too large to be transmitted to the box by the force of friction, which has a maximum value of $\mu_s mg$. In SHM the maximum acceleration is $A\omega^2$, where $A = 0.4$ m is the amplitude and $\omega = 2\pi/0.8 = 7.854\ \mathrm{s}^{-1}$. Thus in a few steps one obtains $A\omega^2 = \mu_s g$ or $\mu_s = 0.4 \times 7.854^2/9.81$ or $\boxed{\mu_s = 2.515}$ (One often hears that $\mu_s \leq 1$, but if the box is glued to the platform this is still physically realistic!)





4. **Neutron in a Box** [15 marks] In 1924, Louis Victor Pierre Raymond duc de Broglie suggested that if light behaves like particles then perhaps particles behave like waves, with a universal relationship between momentum p and wavelength λ given by $p\lambda = h$ (where h is Planck's constant).

   (a) A *neutron* is confined to a one-dimensional box of length $L = 10^{-13}$ m. Taking de Broglie's hypothesis into account, what is the *lowest kinetic energy* it can have? Express your answer in *mega electron volts* (MeV), where 1 MeV $= 1.602 \times 10^{-13}$ J.

   ANSWER: The state with the lowest kinetic energy K is the one with the lowest momentum p, which (by de Broglie's formula) is the one with the longest wavelength λ. The longest wavelength one can fit into a box of length L and still get the required nodes at the endpoints is $\lambda_{\max} = 2L$, giving $p_{\min} = h/2L$ and $K_{\min} = p_{\min}^2/2M_n =$

$h^2/8M_nL^2$. Plugging in $h = 6.6262 \times 10^{-34}$ J-s, $M_n = 1.675 \times 10^{-27}$ kg and $L = 10^{-13}$ m gives $K_{\rm min} = 3.2767 \times 10^{-15}$ J or $\boxed{K_{\rm min} = 2.045 \times 10^{-2} \text{ MeV.}}$

(b) What happens to the neutron's energy if L shrinks by a factor of ten? (This is about the size of a large atomic nucleus.)

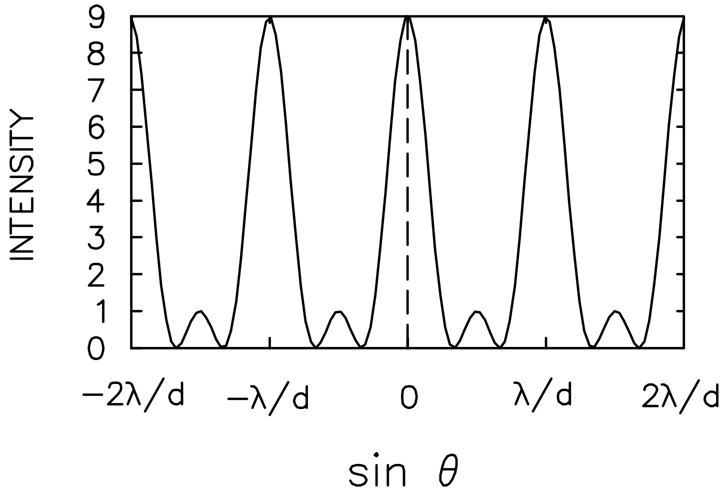
ANSWER: If $\lambda_{\rm max}$ is 10 times smaller, $p_{\rm min}$ is 10 times bigger and $\boxed{K_{\rm min} \text{ is 100 times bigger:}}$ now $K_{\rm min} = 2.045$ MeV.

(c) What can you conclude about the typical binding energy of a neutron in a nucleus?

ANSWER: If the binding energy were not of the order of $\boxed{\text{2 MeV or greater,}}$ then the neutron would have enough kinetic energy to escape, just from the effect of confinement. This result is off by a factor on the order of 2 or 3 for real nuclei, partly because of the 3-dimensional nature of the "box" and partly because the potential created by the strong interaction is not a square well but has "soft sides" allowing the neutron to spread out a bit extra. However, very large nuclei are indeed unstable against the loss of neutrons and protons, a topic to be studied at TRIUMF's new ISAC facility!

5. **Three Slits** [20 marks] Light of wavelength λ falls normally on three extremely narrow slits. Each slit is separated from the next by a distance d. As a result, an interference pattern is formed on a distant screen.

(a) Sketch the intensity on the screen as a function of the sine of the angle from the centre. **ANSWER:**



(b) Give an expression for the angular positions of the principal interference maxima.

ANSWER: $\boxed{d \sin\theta_m = m\lambda}$

(c) Give an expression for θ_1, the smallest angle at which the intensity is zero.

ANSWER: $\frac{2\pi}{\lambda}(d \sin\theta_1) = \frac{2\pi}{3} \implies \boxed{\sin\theta_1 = \frac{\lambda}{3d}}$

(d) Draw a phasor diagram describing what happens at θ_1. **ANSWER:**